



**nothing
nothings**

**perception
does not be**

**perception
percepts**

Damiani • D'Antona • Marra • Palombi

From Combinatorics to Philosophy

The legacy of G.-C. Rota

From Combinatorics to Philosophy: The Legacy of G.-C. Rota provides an assessment of G. -C. Rota's legacy to current international research issues in mathematics, philosophy and computer science.

This volume includes chapters by leading researchers, as well as a number of invited research papers. Rota's legacy connects European and Italian research communities to the USA by providing inspiration to several generations of researchers in combinatorics, philosophy and computer science.

From Combinatorics to Philosophy: The Legacy of G.-C. Rota is of valuable interest to research institutions and university libraries worldwide. This book is also designed for advanced-level students in mathematics, computer science, and philosophy.

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Part I Gian-Carlo Rota, the Man

1	Gianco, my Brother	3
	Ester Gasperoni Rota	
	References	13
2	Remembering Gian-Carlo Rota	17
	Joseph J. Kohn	
	References	21

Part II Gian-Carlo Rota, the Mathematician

3	A Glimpse of Vector Invariant Theory. The Points of View of Weyl, Rota, De Concini and Procesi, and Grosshans	25
	Andrea Brini	
3.1	Introduction	25
3.2	The Characteristic Zero Approach	27
3.2.1	Preliminaries	27
3.2.2	Weyl's Theorem (The Reduction Principle)	28
3.2.3	Absolute Invariants for the Special Linear Group $SL(d)$.	29
3.2.4	Absolute Invariants for the Symplectic Group Sp_{2m}	29
3.2.5	Absolute Invariants for the Special Orthogonal Group $SO(d)$	31
3.2.6	Absolute Invariants for the Orthogonal Group $O(d)$	31
3.3	The Straightening Formulae	32
3.3.1	Standard Young Tableaux	32
3.3.2	Bideterminants	33
3.3.3	Pfaffians	33
3.3.4	Gramians	34
3.4	The E. Pascal Theorems and the Cancellation Laws	35

3.4.1	The E. Pascal Theorem and the Cancellation Law for Scalar Products	36
3.4.2	The E. Pascal Theorem and the Cancellation Law for Symplectic Products	37
3.4.3	The E. Pascal Theorem and the Cancellation Law for Inner Products	38
3.5	The First and the Second Fundamental Theorems	39
3.5.1	The First Fundamental Theorems	39
3.5.2	The Second Fundamental Theorems	40
3.6	Grosshans's Theorem	40
	References	42
4	Partitions of a Finite Partially Ordered Set	45
	Pietro Codara	
4.1	Introduction	45
4.2	Background, and Preliminary Results	46
4.3	Partitions as Sets of Fibres	50
4.4	Partitions as Partially Ordered Sets of Blocks	51
4.5	Partitions Induced by Quasiorders	55
4.6	Further Work	58
	References	59
5	An Algebra of Pieces of Space — Hermann Grassmann to Gian Carlo Rota	61
	Henry Crapo	
5.1	Almost Ten Years Later	61
5.2	Synthetic Projective Geometry	62
5.3	Hermann Grassmann's Algebra	64
5.4	Extensors and Vectors	66
5.5	Reduced Forms	72
5.6	Grassmann-Cayley Algebra, Peano Spaces	73
5.7	Whitney Algebra	75
5.8	Geometric Product	78
5.9	Regressive Product	80
5.10	Higher Order Syzygies	86
5.11	Balls in Boxes	89
	References	90
6	The Eleventh and Twelveth Problems of Rota's Fubini Lectures: from Cumulants to Free Probability Theory	91
	Elvira Di Nardo and Domenico Senato	
6.1	Prologue	91
6.2	The Classical Umbral Calculus	92
6.2.1	Generating Functions	94
6.3	Sequences of Binomial Type, Bell Umbrae and Poisson Processes	96
6.4	Cumulants	99

6.4.1	Singleton Umbra	100
6.4.2	Cumulant Umbra	101
6.5	Applications in Statistics	103
6.5.1	U-Statistics	104
6.5.2	Moments of Sampling Distributions	106
6.5.3	Products of Statistics	109
6.5.4	k -statistics	110
6.5.5	Fast Symbolic Computation of k -statistics	112
6.5.6	Sheppard's Corrections	114
6.6	Sheffer Sequences	117
6.6.1	Abel Polynomials	120
6.6.2	Quasi-free Cumulant Umbra	122
6.6.3	Boolean Cumulants	123
6.7	Free Cumulant Theory	124
	References	127
7	Two Examples of Applied Universal Algebra	131
	Joseph P. S. Kung	
7.1	Prologue	131
7.2	The Bohnenblust–Spitzer Identity	132
7.3	Baxter Algebras	133
7.4	The Standard Baxter Algebra	134
7.5	A Precursor Identity	137
7.6	Abstract Random Variables	141
	References	143
8	On the Euler Characteristic of Finite Distributive Lattices	145
	Emanuele Munarini	
8.1	Introduction	145
8.2	Posets and Distributive Lattices	146
8.3	Euler Characteristic	147
8.4	Uniform Distributions of the Characteristic	151
8.4.1	χ -uniform Lattices	151
8.4.2	Order-preserving Euler Characteristic	152
8.4.3	Rank χ -uniform Lattices	153
8.5	Pseudo-planar Distributive Lattices	154
8.6	Graphs and Hypergraphs	158
8.7	Dual Gödel Lattices	160
8.8	Tree maps	162
	References	165
9	Rota, Probability, Algebra and Logic	167
	Daniele Mundici	
9.1	MV-algebraic States and De Finetti Coherence Criterion	167
9.2	MV-algebraic and C^* -algebraic States After Elliott Classification ..	171
9.3	MV-algebraic σ -states and Carathéodory Probability Theory	176

References	180
10 A Symbolic Treatment of Abel Polynomials	183
Pasquale Petruccio	
10.1 Introduction	183
10.2 Abel polynomials, Lagrange inversion formula and sequences of binomial type	185
10.3 Generalized Abel polynomials	187
10.4 Cumulants and convolutions	191
References	195
 Part III Gian-Carlo Rota, the Philosopher	
11 Ethics in Thought. Gian-Carlo Rota and Philosophy	199
Francesca Bonicalzi	
11.1 Telling/Inventing the Truth	199
11.2 The Winding Streets, and the Straight and Wide Avenue of Precision	202
11.3 The Food of Philosophical Thought and the Medicine of Axiomatics	204
11.4 The Ethics of Knowledge and the Genesis of its Disciplines	206
References	208
12 Indiscrete Variations on Gian-Carlo Rota's Themes	211
Carlo Cellucci	
12.1 Introduction	211
12.2 The Existence of Mathematical Objects	211
12.2.1 Irrelevance of the Existence of Mathematical Objects ...	211
12.2.2 Mathematical Objects as Hypotheses	213
12.2.3 Hypotheses vs. Fictions	215
12.2.4 Existence and Identity	215
12.2.5 The Inexhaustibility of Mathematical Objects	216
12.3 Definition in Mathematics	217
12.3.1 Definition, Description and Analysis of Concepts	217
12.3.2 Definition in Mathematics and in Philosophy	218
12.3.3 The Alleged Circularity of Definitions and Theorems ...	219
12.4 The Notion of Proof	219
12.4.1 Proof as the Opening up of Possibilities	220
12.4.2 Axiomatic Presentation and Gödel's Incompleteness Theorems	221
12.4.3 Are there Definitive Proofs?	221
12.4.4 Proof and Evidence	222
12.5 The Relation of Philosophy of Mathematics to Mathematics	224
12.5.1 The Descriptive Character of the Philosophy of Mathematics	224
12.5.2 The Need for a Drastic Overhaul of Logic	224

12.6	Rota's Place in the Philosophy of Mathematics	225
	References	227
13	On the Courage Needed to Do Phenomenology. Rota and Analytic Philosophy	229
	Albino Lanciani and Claudio Majolino	
13.1	Twilight of an Idol	230
13.2	Three Senses of Phenomenology	234
13.3	Coda	240
	References	240
14	Rota's Philosophical Insights	241
	Massimo Mugnai	
14.1	Introduction	241
14.2	What Kind of Reductionism?	241
14.3	Rota's Objectivism and Husserl's Naturalism	244
14.4	Beyond Classical Logic	245
14.5	Concluding Remarks	248
	References	249
15	"A Minority View". Gian-Carlo Rota's Phenomenological Realism ..	251
	Fabrizio Palombi	
15.1	Considerations on the Problem of Realism	252
15.2	Phenomenological Realism	253
15.3	Fringe Phenomena and State-of-Mind	254
15.4	Psychologistic Misunderstandings	256
15.5	Descriptions, Not Prescriptions	258
	References	259