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This book contains a collection of general mathematical results, formulas, and integrals that occur throughout applications of mathematics. Many of the entries are based on the updated fifth edition of Gradshteyn and Ryzhik's "Tables of Integrals, Series, and Products," though during the preparation of the book, results were also taken from various other reference works. The material has been arranged in a straightforward manner, and for the convenience of the user a quick reference list of the simplest and most frequently used results is to be found in Chapter 0 at the front of the book. Tab marks have been added to pages to identify the twelve main subject areas into which the entries have been divided and also to indicate the main interconnections that exist between them. Keys to the tab marks are to be found inside the front and back covers.

The Table of Contents at the front of the book is sufficiently detailed to enable rapid location of the section in which a specific entry is to be found, and this information is supplemented by a detailed index at the end of the book. In the chapters listing integrals, instead of displaying them in their canonical form, as is customary in reference works, in order to make the tables more convenient to use, the integrands are presented in the more general form in which they are likely to arise. It is hoped that this will save the user the necessity of reducing a result to a canonical form before consulting the tables. Wherever it might be helpful, material has been added explaining the idea underlying a section or describing subtle techniques that are often useful in the application of its results.

Standard notations have been used for functions, and a list of these together with their names and a reference to the section in which they occur or are defined is to be