

"In a very personal writing style, Hodges explores what we do and do not understand about mixed linear models. ... By also discussing mysterious, inconvenient, or plainly wrong results, we simply gain more insight and understanding. This works for me; I have never gained so much (hard to get) insight in so short a time from any other book I have read. I highly recommend it!"

—Håvard Rue, Norwegian University of Science and Technology

"This book is a masterpiece, destined to become a classic. ... There is not presently a unified theory, like that for linear regression, to explain how, why, and when our numerical routines give results that should be questioned, or at least examined further. Even so, this book does the best job I have seen of explaining what can go wrong and what the state of the art is. The subject is important; the writing is excellent; and the examples are compelling. I am excited by the prospect of teaching a course from this book. Its clarity of thought and presentation are exemplary. I recommend it for anyone who fits complicated models."

—Michael Lavine, University of Massachusetts Amherst

Richly Parameterized Linear Models: Additive, Time Series, and Spatial Models Using Random Effects takes a first step in developing a full theory of richly parameterized models, which would allow statisticians to better understand their analysis results. The author examines what is known and unknown about mixed linear models and identifies research opportunities.

Features

- Develops a unified theory for a large class of popular models
- Explains how to overcome some common difficulties in using these models to analyze data
- Covers both Bayesian and non-Bayesian methods
- Includes detailed analyses of many real datasets, with the data and other materials available on the author's website
- Provides standard exercises and open questions at the end of each chapter



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