## **Contents**

List of Figures		ix	
Pref	face approximately approximate	xiii	
PΔR	T I GROUPS		
PART I GROOFS			
1	Preliminaries	3	
1.1	Introduction	3	
1.2	Sets	6	
1.3	The Integers	9	
1.4	Mathematical Induction	14	
1.5	Divisibility, Greatest Common Divisor, Primes, and Unique Factorization	19	
1.6	Modular Arithmetic, Congruences	26	
1.7	Relations	30	
1.8	Functions, the Pigeonhole Principle, and Binary Operations	34	
2	Groups: A Beginning	43	
2.1	What is a Group?	43	
2.2	Visualizing Groups	52	
2.3	More Examples of Groups and Some Basic Facts	56	
2.4	Subgroups	64	
2.5	Cyclic Groups are Our Friends	72	
3	Groups: There's More	81	
3.1	Groups of Permutations	81	
3.2	Isomorphisms and Cayley's Theorem	89	
3.3	Cosets, Lagrange's Theorem, and Normal Subgroups	93	
3.4	Building New Groups from Old, I: Quotient or Factor Groups G/H	98	
3.5	Group Homomorphism	102	
3.6	Building New Groups from Old, II: Direct Product of Groups	108	
3.7	Group Actions	114	
4	Applications and More Examples of Groups	124	
4.1	Public-Key Cryptography	124	
4.2	Chemistry and the Finite Fourier Transform	129	
4.3	Groups and Conservation Laws in Physics	135	
4.4	Puzzles	142	
4.5	Small Groups	146	

## **PART II RINGS**

5	Rings: A Beginning	157
5.1	Introduction	157
5.2	What is a Ring?	158
5.3	Integral Domains and Fields are Nicer Rings	166
5.4	Building New Rings from Old: Quotients and Direct Sums of Rings	173
5.5	Polynomial Rings	180
5.6	Quotients of Polynomial Rings	185
6	Rings: There's More	189
6.1	Ring Homomorphisms	189
6.2	The Chinese Remainder Theorem	193
6.3	More Stories about $F[x]$ Including Comparisons with $\mathbb{Z}$	198
6.4	Field of Fractions or Quotients	202
7	Vector Spaces and Finite Fields	206
7.1	Matrices and Vector Spaces over Arbitrary Fields and Rings like $\mathbb Z$	206
7.2	Linear Functions or Mappings	218
7.3	Determinants	224
7.4	Extension Fields: Algebraic versus Transcendental	229
7.5	Subfields and Field Extensions of Finite Fields	233
7.6	Galois Theory for Finite Fields	239
8	Applications of Rings	244
8.1	Random Number Generators	244
8.2	Error-Correcting Codes	256
8.3	Finite Upper Half Planes and Ramanujan Graphs	265
8.4	Eigenvalues, Random Walks on Graphs, and Google	272
8.5	Elliptic Curve Cryptography	282
Refe	rences	299
Index sould state the state of		305