

This book is an introduction to the general theory of second order parabolic differential equations, which model many important, time-dependent physical systems. It studies the existence, uniqueness, and regularity of solutions to a variety of problems with Dirichlet boundary conditions and general linear and nonlinear boundary conditions by means of a priori estimates. The first seven chapters give a description of the linear theory and are suitable for a graduate course on partial differential equations. The last eight chapters cover the nonlinear theory for smooth solutions. They include much of the author's research and are aimed at researchers in the field. A unique feature is the emphasis on time-varying domains.

*"In the reviewer's opinion the author of this nicely written book has succeeded very well in his goal that 'this book was to create a companion volume to Elliptic Partial Differential Equations of Second Order by David Gilbarg and Neil S Trudinger'."*

**Mathematical Reviews**

*"The book provides an essentially self-contained exposition of the theory of second order parabolic partial differential equations."*

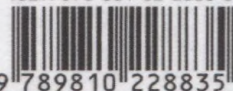
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