

Contents

Preface to the Third Edition	ix
To Students	xi
1. The Physical Origins of Partial Differential Equations	1
1.1 PDE Models	2
1.2 Conservation Laws	12
1.3 Diffusion	29
1.4 Diffusion and Randomness	38
1.5 Vibrations and Acoustics	49
1.6 Quantum Mechanics*	57
1.7 Heat Conduction in Higher Dimensions.....	60
1.8 Laplace's Equation	66
1.9 Classification of PDEs	72
2. Partial Differential Equations on Unbounded Domains	79
2.1 Cauchy Problem for the Heat Equation	79
2.2 Cauchy Problem for the Wave Equation	87
2.3 Well-Posed Problems.....	92
2.4 Semi-Infinite Domains.....	96
2.5 Sources and Duhamel's Principle	101
2.6 Laplace Transforms	106
2.7 Fourier Transforms	117
3. Orthogonal Expansions	127
3.1 The Fourier Method	127
3.2 Orthogonal Expansions.....	131

3.3 Classical Fourier Series	145
4. Partial Differential Equations on Bounded Domains	155
4.1 Overview of Separation of Variables	156
4.2 Sturm–Liouville Problems	167
4.3 Generalization and Singular Problems	180
4.4 Laplace’s Equation	186
4.5 Cooling of a Sphere	198
4.6 Diffusion in a Disk	202
4.7 Sources on Bounded Domains	207
4.8 Poisson’s Equation*	216
5. Applications in the Life Sciences	229
5.1 Age-Structured Models	229
5.2 Traveling Waves Fronts	238
5.3 Equilibria and Stability	245
6. Numerical Computation of Solutions	257
6.1 Finite Difference Approximations	258
6.2 Explicit Scheme for the Heat Equation	260
6.3 Laplace’s Equation	268
6.4 Implicit Scheme for the Heat Equation	273
Appendix A. Differential Equations	279
References	285
Index	287