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Theory of Translation Closedness for Time Scales

With Applications in Translation Functions and Dynamic Equations

This monograph establishes a theory of classification and translation closedness of time scales, a topic that was first studied by S. Hilger in 1988 to unify continuous and discrete analysis. The authors develop a theory of translation function on time scales that contains (piecewise) almost periodic functions, (piecewise) almost automorphic functions and their related generalization functions (e.g., pseudo almost periodic functions, weighted pseudo almost automorphic functions, and more). Against the background of dynamic equations, these function theories on time scales are applied to study the dynamical behavior of solutions for various types of dynamic equations on hybrid domains, including evolution equations, discontinuous equations and impulsive integro-differential equations.

The theory presented allows many useful applications, such as in the Nicholson's blowflies model; the Lasota-Ważewska model; the Keynesian-Cross model; in those realistic dynamical models with a more complex hybrid domain, considered under different types of translation closedness of time scales; and in dynamic equations on mathematical models which cover neural networks. This book provides readers with the theoretical background necessary for accurate mathematical modeling in physics, chemical technology, population dynamics, biotechnology and economics, neural networks, and social sciences.

ISBN 978-3-030-38643-6



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