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some aspects of a given reality within the framework of a mathematical apparatus that provides us with a means for exploring the properties of that reality mirrored in the model.

This book is about the ways and means of constructing "good" models of reality, the properties of such models, the means for encoding specific realities into definite formal systems and the procedures for interpreting the properties of the formal system in terms of the given real world situation. In short, we are interested in the ways of model-making. Before embarking upon a more detailed account of what we mean by the terms *model*, *encoding*, *formal system*, and so forth, it's necessary to examine some basic epistemological and operational issues lying at the heart of what we shall term the *theory of models*.

According to the great nineteenth-century British physicist Maxwell, "the success of any physical investigation depends upon the judicious selection of what is to be observed as of primary importance."⁵ This view suggests the notion that what constitutes one's reality depends upon one's capacity for observation. Here we adopt the position that since natural phenomena impinge upon our consciousness only through instruments of observation, and, to paraphrase Maxwell, "the success of any modeling venture depends upon a judicious selection of observables and means for encapsulating these variables within the framework of convenient formal mathematical systems."

As noted by Rosen, in dealing with the idea of a natural system, we must necessarily touch on some basic philosophical questions of both an ontological and epistemological character. This is unavoidable in any case