



THINKING MATHEMATICALLY

is for

- anyone who has the feeling that mathematical thinking involves more than glorified arithmetic, but is not sure in what way;
- anyone who has been intrigued by contact with some mathematical ideas, and wishes to become involved in this kind of thinking;
- anyone who has wondered what lies at the core of mathematical knowledge;
- anyone concerned enough to enquire beneath the surface of mathematics;
- anyone who has responsibility for fostering mathematical thinking in others.

THINKING MATHEMATICALLY

unfolds the processes which lie at the heart of mathematics. It demonstrates how to encourage, develop and foster the processes which seem to come naturally to mathematicians. Intensely practical, it demands that the reader participate in each question posed, so that the subsequent discussion speaks to immediate experience. In this way, a deep seated awareness of the nature of mathematical thinking can grow.

THINKING MATHEMATICALLY

offers 100 questions and, in addition, suggests to readers how to find and pose questions of their own. In this revised edition special care has been taken in the presentation of the text and the authors have clarified some of their previous material.



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Introduction

1. Everyone can start

Specializing

Warehouse

Paper Strip

Palindromes

Generalizing

Writing yourself notes

Patchwork

Chessboard Squares

Review and preview

2. Phases of work

Three phases

The Entry phase

Tethered Goat

Entry 1: What do I KNOW?

Ladies Luncheon

Entry 2: What do I WANT?

Fractious

Envelopes

Entry 3: What can I INTRODUCE?

Cubes Cubed

Quick and Toasty

Entry summarized

The Attack phase

The Review phase

Review 1: CHECK the resolution

Review 2: REFLECT on the key ideas and key moments

Review 3: EXTEND to a wider context

Chessboard Rectangles

Practising Review

Creepy Crawlies

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