

LONDON MATHEMATICAL SOCIETY LECTURE NOTE SERIES

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Higher Operads, Higher Categories

Edited by Tom Leinster

Higher-dimensional category theory is the study of n -categories, operads, braided monoidal categories and other such exotic structures. It draws its inspiration from areas as diverse as topology, quantum algebra, mathematical physics, logic and theoretical computer science.

The heart of this book is the language of generalized operads. This is as natural and transparent a language for higher category theory as the language of sheaves is for algebraic geometry, or vector spaces for linear algebra. It is introduced carefully, then used to give simple descriptions of a variety of higher categorical structures. In particular, one possible definition of n -category is discussed in detail, and some common aspects of other possible definitions are established.

This is the first book on the subject and lays its foundations. It will appeal to both graduate students and established researchers who wish to become acquainted with this modern branch of mathematics.

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