

[The book] starts with an account of the definitions, and a development of the homotopy theory of model categories. This is probably the first time in which the important notion of cofibrant generation has appeared in a book, and the consideration of the 2-category of model categories and Quillen adjunctions is another interesting feature.

—*Bulletin of the London Mathematical Society*

Model categories are used as a tool for inverting certain maps in a category in a controllable manner. As such, they are useful in diverse areas of mathematics. The list of such areas is continually growing.

This book is a comprehensive study of the relationship between a model category and its homotopy category. The author develops the theory of model categories, giving a careful development of the main examples. One highlight of the theory is a proof that the homotopy category of any model category is naturally a closed module over the homotopy category of simplicial sets.

Little is required of the reader beyond some category theory and set theory, which makes the book accessible to advanced graduate students. The book begins with the basic theory of model categories and proceeds to a careful exposition of the main examples, using the theory of cofibrantly generated model categories. It then develops the general theory more fully, showing in particular that the homotopy category of any model category is a module over the homotopy category of simplicial sets, in an appropriate sense. This leads to a simplification and generalization of the loop and suspension functors in the homotopy category of a pointed model category. The book concludes with a discussion of the stable case, where the homotopy category is triangulated in a strong sense and has a set of small weak generators.

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