'In recent years the methods of homotopy theory have seen increasingly wide applications in mathematics, and the framework of abstract homotopy theory has been found to be an important lens through which to view many mathematical structures. This book offers a single, self-contained place to learn about the extensive modern facets of abstract homotopy theory. Readers will appreciate Cisinski's thoughtful choice of details and his carefully articulated philosophical point of view. This is an excellent resource for mathematicians experiencing first contact with the subject and for more seasoned researchers in the area.'

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'This book is a window into the new field of mathematics emerging from the convergence of category theory and homotopy theory. It widens and deepens the extension from category theory into quasi-categories with the addition of a new theory of presheaves inspired by type theory and a new theory of localisation; it further proposes an extension of homotopical algebra to quasi-categories, offers new applications, and brings important simplification to earlier works. It is an excellent introduction to the subject and may be used for an advanced course.'

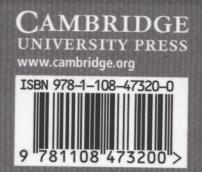
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'Denis-Charles Cisinski offers a masterful introduction to the world of ∞ -categories, illustrating the necessary intuition all throughout. A complete and clear exposition of the foundations leads naturally to a full course teaching us how to handle all aspects of homotopical algebra within the theory.'

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