

Kinetic Theory of Gases and Plasmas

by

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Kinetic theory provides the link between the non-equilibrium statistical mechanics of many-particle systems and macroscopic or phenomenological physics.

This volume deals with the derivation of kinetic equations, their limitations and generalizations, and with the applications of kinetic theory to physical phenomena and the calculation of transport coefficients.

The book is divided into twelve chapters which discuss a wide range of topics such as balanced equations, the Klimontovich, Vlasov-Maxwell, and Boltzmann equations, Chapman-Enskog theory, the kinetic theory of plasmas, B.G.K. models, linear response theory, Brownian motion and renormalized kinetic theory.

Each chapter is concluded with exercises, which not only enable the readers to test their understanding of the theory, but also present additional examples which complement the text.

Audience

This volume will be of interest to researchers whose work involves kinetic theory. It can also be recommended as a textbook for advanced undergraduate and graduate students.

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