

Contents

Introduction	9
Chapter 1. Stepped Semi-Markov Processes	17
1.1. Random sequence	17
1.2. Markov chain	20
1.3. Two-dimensional Markov chain	25
1.4. Semi-Markov process	29
1.5. Stationary distributions	32
Chapter 2. Sequences of First Exit Times and Regeneration Times	37
2.1. Basic maps	38
2.2. Markov times	42
2.3. Deducing sequences	46
2.4. Correct exit and continuity	55
2.5. Time of regeneration	63
Chapter 3. General Semi-Markov Processes	71
3.1. Definition of a semi-Markov process	72
3.2. Transition function of a SM process	79
3.3. Operators and SM walk	82
3.4. Operators and SM process	91
3.5. Criterion of Markov property for SM processes	103
3.6. Intervals of constancy	110
Chapter 4. Construction of Semi-Markov Processes using Semi-Markov Transition Functions	115
4.1. Realization of an infinite system of pairs	116
4.2. Extension of a measure	121

4.3. Construction of a measure	124
4.4. Construction of a projective system of measures	127
4.5. Semi-Markov processes	133
Chapter 5. Semi-Markov Processes of Diffusion Type	137
5.1. One-dimensional semi-Markov processes of diffusion type	138
5.1.1. Differential equation	138
5.1.2. Construction SM process	143
5.1.3. Some properties of the process	159
5.2. Multi-dimensional processes of diffusion type	168
5.2.1. Differential equations of elliptic type	168
5.2.2. Neighborhood of arbitrary form	171
5.2.3. Neighborhood of spherical form	178
5.2.4. Characteristic operator	188
Chapter 6. Time Change and Semi-Markov Processes	197
6.1. Time change and trajectories	198
6.2. Intrinsic time and traces	206
6.3. Canonical time change	211
6.4. Coordination of function and time change	223
6.5. Random time changes	228
6.6. Additive functionals	233
6.7. Distribution of a time run along the trace	242
6.8. Random curvilinear integrals	252
6.9. Characteristic operator and integral	264
6.10. Stochastic integral	268
6.10.1. Semi-martingale and martingale	268
6.10.2. Stochastic integral	275
6.10.3. Ito-Dynkin's formula	277
Chapter 7. Limit Theorems for Semi-Markov Processes	281
7.1. Weak compactness and weak convergence	281
7.2. Weak convergence of semi-Markov processes	289
Chapter 8. Representation of a Semi-Markov Process as a Transformed Markov Process	299
8.1. Construction by operator	300
8.2. Comparison of processes	302
8.3. Construction by parameters of Lévy formula	307
8.4. Stationary distribution	311

Chapter 9. Semi-Markov Model of Chromatography	325
9.1. Chromatography	326
9.2. Model of liquid column chromatography	328
9.3. Some monotone Semi-Markov processes	332
9.4. Transfer with diffusion	337
9.5. Transfer with final absorption	346
Bibliography	361
Index	369

It is well known that the class of Markov processes, having a rather limited applicability in the theory of stochastic processes, is the most important one. In the 1940s, some practical problems of the theory of stochastic processes required a further extension to look for an appropriate class of stochastic processes. It is naturally Markov models in these problems that are considered. The main conclusion about exponential distributions of waiting times in the theory of Markov processes is that in order to reduce this limitation R. Lévy (1948) proposed the class of stepped processes. For these processes the Markov property is fulfilled at a fixed instant, but it is not fulfilled in general cases, but the former Markov property is valid respect to some random times, for example, jump times of their trajectories. Therefore, such a process is not a Markov process, although it inherits some important properties of Markov processes.

Recently the theory of stepped semi-Markov processes has been developed more intensively. An extensive bibliography of corresponding works was compiled by Dragalin [DRA 73], Korobukhin [KOR 74], and Turbin [TUR 74]. Korobukhin and Turbin [KOR 74] also compiled a list of references [CHE 73], Nollau [NOL 80]. The list of works devoted to the theory of stepped semi-Markov processes is very long (see Kovalenko [KOV 85], Kovalenko and Korobukhin [KK 85], etc.).

We will consider in this chapter the class of stepped semi-Markov processes with trajectories in a metric space. We will consider the case when the trajectories have jumps from the left at any point of the time axis. We will consider the case when the trajectories are stepped semi-Markov processes (in the general sense), have the Markov property at any instant of time, and at any intrinsic Markov time such as the first exit time from a set. We will consider the case when the trajectories are stepped semi-Markov processes, and also all strong Markov processes, and also all strong Markov processes.

Apparently for the first time the class of stepped semi-Markov processes [HAR 74] were defined in [HAR 74] as a class of processes with independent return times.