

Canonical systems occupy a central position in the spectral theory of second order differential operators. They may be used to realize arbitrary spectral data, and the classical operators such as Schrödinger, Jacobi, Dirac, and Sturm-Liouville equations can be written in this form. 'Spectral Theory of Canonical Systems' offers a self-contained and detailed introduction to this theory. Techniques to construct self-adjoint realizations in suitable Hilbert spaces, a modern treatment of de Branges spaces, and direct and inverse spectral problems are discussed.

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