COMBINATORIAL MATHEMATICS

- · Covers enumeration, graphs, sets, and methods
- · Assumes no prior exposure to combinatorics
- · Suitable for a one-year sequence, or as a one-semester introduction leading to advanced courses
- · Contains more than 2200 exercises at various levels
- · Complete solutions manual available for instructors at www.cambridge.org/west
- Includes more than 2300 references and many exercises, theorems, and proofs that have not previously appeared in textbooks
- · Includes both classical results and more recent developments

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"A comprehensive and comprehensible treatment of combinatorial mathematics – what the author intended, and more – a valuable resource on the subject. It may well stimulate the addition to the university curriculum of courses in this area. Borrowing advice from the past, my recommendation for anyone wanting a definitive book on combinatorics is 'Go West, my friend, go West!'"

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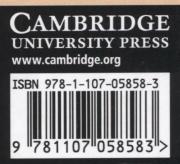
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"This is a great book by a great writer. It covers the main topics of combinatorics, is well written, and presents many beautiful ideas and techniques. It is very helpful to students, teachers, and researchers. I would like to have this book on my desk."

ALEXANDR KOSTOCHKA, University of Illinois

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