

Architecture of Mathematics describes the logical structure of Mathematics from its foundations to its real-world applications. It describes the many interweaving relationships between different areas of mathematics and its practical applications, and as such provides unique reading for professional mathematicians and nonmathematicians alike. This book can be a very important resource both for the teaching of mathematics and as a means to outline the research links between different subjects within and beyond the subject.

Features

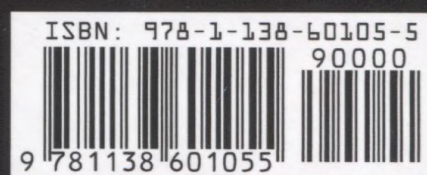
- All notions and properties are introduced logically and sequentially, to help the reader gradually build understanding.
- Focuses on illustrative examples that explain the meaning of mathematical objects and their properties.
- Suitable as a supplementary resource for teaching undergraduate mathematics, and as an aid to interdisciplinary research.
- Forming the reader's understanding of mathematics as a unified science, the book helps to increase his general mathematical culture.

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MATHEMATICS



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