

Theoretical and Mathematical Physics

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Theoretical Fluid Dynamics

This textbook gives an introduction to fluid dynamics based on flows for which analytical solutions exist, like individual vortices, vortex streets, vortex sheets, accretions disks, wakes, jets, cavities, shallow water waves, bores, tides, linear and non-linear free-surface waves, capillary waves, internal gravity waves and shocks.

Advanced mathematical techniques ("calculus") are introduced and applied to obtain these solutions, mostly from complex function theory (Schwarz-Christoffel theorem and Wiener-Hopf technique), exterior calculus, singularity theory, asymptotic analysis, the theory of linear and nonlinear integral equations and the theory of characteristics.

Many of the derivations, so far contained only in research journals, are made available here to a wider public.

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