

Sources and Studies in the History of Mathematics and Physical Sciences

Jacques Sesiano

Magic Squares

Their History and Construction from Ancient Times to AD 1600

The science of magic squares witnessed an important development in the Islamic world during the Middle Ages, with a great variety of construction methods being created and ameliorated. The initial step was the translation, in the ninth century, of an anonymous Greek text containing the description of certain highly developed arrangements, no doubt the culmination of ancient research on magic squares.

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