

In 1971, Alexey Stakhov was elected to the post of the Head of the Department of Information and Measurement Technology of the Taganrog Radio Engineering Institute, where he worked for 7 years (1971–1977). During “Taganrog period”, Stakhov defended his **Doctoral Dissertation** (1972), received the academic title of the Professor (1974) and wrote his first book *Introduction to the Algorithmic Theory of Measurement* (1977) (“Soviet Radio”, Moscow).

In January 1976, according to decision of the Soviet Ministry of Higher Education, Prof. Stakhov was directed on a 2-month scientific trip to Austria for scientific work at leading Austrian Universities (in particular, at Vienna and Graz Technical Universities). On the concluding stage of his stay in Austria, Stakhov made the speeches about his scientific direction on the Graz Technical University and then on the joint Session of the Austrian Cybernetics and Computer Societies (Vienna). His speeches aroused great interest of the famous Austrian scientists. 4 leading Austrian scientists in his supporting letters highly evaluated deep scientific ideas of Prof. Stakhov.

The present 3-volume book is intended for a wide audience, including teachers of high schools, students of colleges and universities and scientists in mathematics, theoretical physics and computer science.

**The excerpt of the academician Yuri Mitropolsky, the head of the Ukrainian mathematical school, from the letter to Prof. Stakhov:**

“Stakhov’s publications are closing a cycle of his long-term research on the creation of a new direction in mathematics, the ‘Mathematics of Harmony’.

One may wonder what place in the general theory of mathematics Stakhov’s work may have. It seems to me that it is in the last few centuries as Nikolay Lobachevsky said, ‘**Mathematicians have turned all their attention to the advanced parts of analytics and neglected the origins of Mathematics, and not willing to dig the field that has already been harvested by them and left behind.**’ As a result, it has created a gap between ‘Elementary Mathematics’ — the bases of modern mathematical education — and ‘Advanced Mathematics’.

In my opinion, the Mathematics of Harmony, created by Professor Stakhov, fills that gap. Mathematics of Harmony is a huge theoretical contribution to the development of ‘Elementary Mathematics’, and as such it should be considered as having great importance for mathematical education.”

**Yuri Mitropolsky**

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