

Based on a course in the theory of statistics this text presents the role of likelihood in a whole range of statistical problems, from a simple comparison of two accident rates to complex studies requiring generalized linear or semiparametric modelling. The emphasis is that the likelihood is not simply a device to produce an estimate, but more importantly it is a tool for modelling.

The book generally takes an informal approach, where most of the important results are established using heuristic arguments and motivated with realistic examples. With the currently available computing power, examples are not contrived to allow a closed analytical solution, and the book can concentrate on the statistical aspects of the data modelling. In addition to classical likelihood theory, the book covers many modern topics such as generalized linear models and mixed models, nonparametric smoothing, robustness, the EM algorithm and empirical likelihood.

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'This is a splendid book with its contents thoroughly covering all likelihood...Statements are firm, and explanations are full and clear. This book may be used as a reference work. It is strongly recommended as an academic library volume, and individually for statistics lecturers, advanced students, and researchers.' The Mathematical Gazette

'To those of us to whom it is a continuing irritation to be told that there are only two kinds of statisticians, frequentist and Bayesian, this book will come as an enormous relief... a remarkable book, which deserves the widest distribution; I hope it will gain many converts to the likelihood school.' Biometrics

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OXFORD
UNIVERSITY PRESS

www.oup.com

ISBN 978-0-19-967122-9



9 780199 671229

1	Introduction	1
1.1	Prototype of statistical problems	1
1.2	Statistical problems and their models	3
1.3	Statistical uncertainty: inevitable controversies	6
1.4	The emergence of statistics	8
1.5	Fisher and the third way	14
1.6	Exercises	19
2	Elements of likelihood inference	21
2.1	Classical definition	21
2.2	Examples	24
2.3	Combining likelihoods	27
2.4	Likelihood ratio	29
2.5	Maximum and curvature of likelihood	30
2.6	Likelihood-based intervals	35
2.7	Standard error and Wald statistic	41
2.8	Invariance principle	43
2.9	Practical implications of invariance principle	45
2.10	Exercises	48
3	More properties of likelihood	53
3.1	Sufficiency	53
3.2	Minimal sufficiency	55
3.3	Multiparameter models	58
3.4	Profile likelihood	61
3.5	Calibration in multiparameter case	64
3.6	Exercises	67
4	Basic models and simple applications	73
4.1	Binomial or Bernoulli models	73
4.2	Binomial model with under- or overdispersion	76
4.3	Comparing two proportions	78
4.4	Poisson model	82
4.5	Poisson with overdispersion	84
4.6	Traffic deaths example	86
4.7	Aspirin data example	87

4.8	Continuous data	89
4.9	Exponential family	95
4.10	Box–Cox transformation family	102
4.11	Location-scale family	104
4.12	Exercises	107
5	Frequentist properties	117
5.1	Bias of point estimates	117
5.2	Estimating and reducing bias	119
5.3	Variability of point estimates	123
5.4	Likelihood and P-value	125
5.5	CI and coverage probability	128
5.6	Confidence density, CI and the bootstrap	131
5.7	Exact inference for Poisson model	134
5.8	Exact inference for binomial model	139
5.9	Nuisance parameters	140
5.10	Criticism of CIs	142
5.11	Exercises	145
6	Modelling relationships: regression models	149
6.1	Normal linear models	150
6.2	Logistic regression models	154
6.3	Poisson regression models	157
6.4	Nonnormal continuous regression	160
6.5	Exponential family regression models	163
6.6	Deviance in GLM	166
6.7	Iterative weighted least squares	174
6.8	Box–Cox transformation family	178
6.9	Location-scale regression models	181
6.10	Exercises	187
7	Evidence and the likelihood principle*	193
7.1	Ideal inference machine?	193
7.2	Sufficiency and the likelihood principles	194
7.3	Conditionality principle and ancillarity	196
7.4	Birnbaum’s theorem	197
7.5	Sequential experiments and stopping rule	199
7.6	Multiplicity	204
7.7	Questioning the likelihood principle	206
7.8	Exercises	211
8	Score function and Fisher information	213
8.1	Sampling variation of score function	213
8.2	The mean of $S(\theta)$	215
8.3	The variance of $S(\theta)$	216
8.4	Properties of expected Fisher information	219

8.5	Cramér–Rao lower bound	221
8.6	Minimum variance unbiased estimation*	223
8.7	Multiparameter CRLB	226
8.8	Exercises	228
9	Large-sample results	231
9.1	Background results	231
9.2	Distribution of the score statistic	235
9.3	Consistency of MLE for scalar θ	238
9.4	Distribution of MLE and the Wald statistic	241
9.5	Distribution of likelihood ratio statistic	243
9.6	Observed versus expected information*	244
9.7	Proper variance of the score statistic*	247
9.8	Higher-order approximation: magic formula*	247
9.9	Multiparameter case: $\theta \in R^p$	256
9.10	Examples	259
9.11	Nuisance parameters	264
9.12	χ^2 goodness-of-fit tests	268
9.13	Exercises	270
10	Dealing with nuisance parameters	273
10.1	Inconsistent likelihood estimates	274
10.2	Ideal case: orthogonal parameters	276
10.3	Marginal and conditional likelihoods	278
10.4	Comparing Poisson means	281
10.5	Comparing proportions	283
10.6	Modified profile likelihood*	286
10.7	Estimated likelihood	292
10.8	Exercises	294
11	Complex data structures	297
11.1	ARMA models	297
11.2	Markov chains	299
11.3	Replicated Markov chains	302
11.4	Spatial data	305
11.5	Censored/survival data	309
11.6	Survival regression models	314
11.7	Hazard regression and Cox partial likelihood	316
11.8	Poisson point processes	320
11.9	Replicated Poisson processes	324
11.10	Discrete time model for Poisson processes	331
11.11	Exercises	335
12	EM Algorithm	341
12.1	Motivation	341
12.2	General specification	342

12.3	Exponential family model	344
12.4	General properties	348
12.5	Mixture models	349
12.6	Robust estimation	352
12.7	Estimating infection pattern	354
12.8	Mixed model estimation*	356
12.9	Standard errors	359
12.10	Exercises	362
13	Robustness of likelihood specification	365
13.1	Analysis of Darwin's data	365
13.2	Distance between model and the 'truth'	367
13.3	Maximum likelihood under a wrong model	370
13.4	Large-sample properties	372
13.5	Comparing working models with the AIC	375
13.6	Deriving the AIC	379
13.7	Exercises	383
14	Estimating equations and quasi-likelihood	385
14.1	Examples	387
14.2	Computing $\hat{\beta}$ in nonlinear cases	390
14.3	Asymptotic distribution	393
14.4	Generalized estimating equation	395
14.5	Robust estimation	398
14.6	Asymptotic Properties	404
15	Empirical likelihood	409
15.1	Profile likelihood	409
15.2	Double-bootstrap likelihood	413
15.3	BC_a bootstrap likelihood	415
15.4	Exponential family model	418
15.5	General cases: M-estimation	420
15.6	Parametric versus empirical likelihood	422
15.7	Exercises	424
16	Likelihood of random parameters	425
16.1	The need to extend the likelihood	425
16.2	Statistical prediction	427
16.3	Defining extended likelihood	429
16.4	Exercises	433
17	Random and mixed effects models	435
17.1	Simple random effects models	436
17.2	Normal linear mixed models	439
17.3	Estimating genetic value from family data*	442
17.4	Joint estimation of β and b	444

17.5	Computing the variance component via $\hat{\beta}$ and $\hat{b}$	445
17.6	Examples	448
17.7	Extension to several random effects	452
17.8	Generalized linear mixed models	458
17.9	Exact likelihood in GLMM	460
17.10	Approximate likelihood in GLMM	462
17.11	Exercises	469
18	Nonparametric smoothing	473
18.1	Motivation	473
18.2	Linear mixed models approach	477
18.3	Imposing smoothness using random effects model	479
18.4	Penalized likelihood approach	481
18.5	Estimate of f given σ^2 and σ_b^2	482
18.6	Estimating the smoothing parameter	485
18.7	Prediction intervals	489
18.8	Partial linear models	489
18.9	Smoothing nonequispaced data*	490
18.10	Non-Gaussian smoothing	492
18.11	Nonparametric density estimation	497
18.12	Nonnormal smoothness condition*	500
18.13	Exercises	501
	Bibliography	503
	Index	515