

Presents a unified approach to parametric estimation, confidence intervals, hypothesis testing, and statistical modeling, which are uniquely based on the likelihood function

This book addresses mathematical statistics for upper-undergraduates and first year graduate students, tying chapters on estimation, confidence intervals, hypothesis testing, and statistical models together to present a unifying focus on the likelihood function. It also emphasizes the important ideas in statistical modeling, such as sufficiency, exponential family distributions, and large sample properties. *Mathematical Statistics: An Introduction to Likelihood Based Inference* makes advanced topics accessible and understandable and covers many topics in more depth than typical mathematical statistics textbooks. It includes numerous examples, case studies, a large number of exercises ranging from drill and skill to extremely difficult problems, and many of the important theorems of mathematical statistics along with their proofs.

In addition to the connected chapters mentioned above, *Mathematical Statistics* covers likelihood based estimation, with emphasis on multidimensional parameter spaces and range dependent support. It also includes a chapter on confidence intervals, which contains examples of exact confidence intervals along with the standard large sample confidence intervals based on the MLE's and bootstrap confidence intervals. There is also a chapter on parametric statistical models featuring sections on non-iid observations, linear regression, logistic regression, Poisson regression, and linear models. This book:

- Prepares students with the tools needed to be successful in their future work in statistics data science
- Includes practical case studies including real-life data collected from Yellowstone National Park, the Donner party, and the Titanic voyage
- Emphasizes the important ideas to statistical modeling, such as sufficiency, exponential family distributions, and large sample properties
- Includes sections on Bayesian estimation and credible intervals
- Features examples, problems, and solutions

Mathematical Statistics: An Introduction to Likelihood Based Inference is an ideal textbook for upper-undergraduate and graduate courses in probability, mathematical statistics, and statistical inference.

Richard J. Rossi, PhD, is Director of the Statistics Program and Co-Director of the Data Science Program at Montana Tech of The University of Montana, in Butte, MT. He acted as President of the Montana Chapter of the American Statistical Association in 2001 and as Associate Editor for Biometrics from 1997-2000. Dr. Rossi is a member of the American Mathematical Society, the Institute of Mathematical Statistics, and the American Statistical Association.

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ISBN 978-1-118-77104-4



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