

The long-anticipated revision of this well-liked textbook offers many new additions. In the twenty-five years since the original version of this book was published, much has happened in dynamical systems. Mandelbrot and Julia sets were barely ten years old when the first edition appeared, and most of the research involving these objects then centered around iterations of quadratic functions. This research has expanded to include all sorts of different types of functions, including higher-degree polynomials, rational maps, exponential and trigonometric functions, and many others. Several new sections in this edition are devoted to these topics.


The area of dynamical systems covered in **A First Course in Chaotic Dynamical Systems: Theory and Experiment, Second Edition** is quite accessible to students and also offers a wide variety of interesting open questions for students at the undergraduate level to pursue. The only prerequisite for students is a one-year calculus course (no differential equations required); students will easily be exposed to many interesting areas of current research. This course can also serve as a bridge between the low-level, often non-rigorous calculus courses, and the more demanding higher-level mathematics courses.

### Features

- More extensive coverage of fractals, including objects like the Sierpinski carpet and others that appear as Julia sets in the later sections on complex dynamics, as well as an actual chaos “game.”
- More detailed coverage of complex dynamical systems like the quadratic family and the exponential maps.
- New sections on other complex dynamical systems such as rational maps.
- A number of new and expanded computer experiments for students to perform.

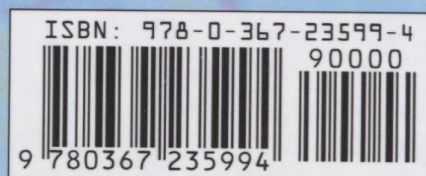
### About the Author

**Robert L. Devaney** is currently professor of mathematics at Boston University. He earned his PhD from the University of California at Berkeley under the direction of Stephen Smale. He taught at Northwestern University and Tufts University before coming to Boston University in 1980. His main area of research is dynamical systems, primarily complex analytic dynamics, but also more general ideas about chaotic dynamical systems. Lately, he has become intrigued with the incredibly rich topological aspects of dynamics, including such things as indecomposable continua, Sierpinski curves, and Cantor bouquets.

 **CRC Press**  
Taylor & Francis Group  
an informa business  
[www.crcpress.com](http://www.crcpress.com)

CRC Press titles are available as eBook editions in a range of digital formats

MATHEMATICS





6.3	The Period-Doubling Bifurcation . . . . .	69
6.4	Experiment: The Transition to Chaos . . . . .	73
<b>7</b>	<b>The Quadratic Family</b>	<b>79</b>
7.1	The Case $c = -2$ . . . . .	79
7.2	The Case $c < -2$ . . . . .	81
7.3	The Cantor Middle-Thirds Set . . . . .	85
<b>8</b>	<b>Transition to Chaos</b>	<b>91</b>
8.1	The Orbit Diagram . . . . .	91
8.2	The Period-Doubling Route to Chaos . . . . .	96
8.3	Experiment: Windows in the Orbit Diagram . . . . .	97
<b>9</b>	<b>Symbolic Dynamics</b>	<b>105</b>
9.1	Itineraries . . . . .	105
9.2	The Sequence Space . . . . .	106
9.3	The Shift Map . . . . .	111
9.4	Conjugacy . . . . .	113
<b>10</b>	<b>Chaos</b>	<b>121</b>
10.1	Three Properties of a Chaotic System . . . . .	121
10.2	Other Chaotic Systems . . . . .	127
10.3	Manifestations of Chaos . . . . .	132
10.4	Experiment: Feigenbaum's Constant . . . . .	134
<b>11</b>	<b>Sharkovsky's Theorem</b>	<b>139</b>
11.1	Period 3 Implies Chaos . . . . .	139
11.2	Sharkovsky's Theorem . . . . .	142
11.3	The Period-3 Window . . . . .	147
11.4	Subshifts of Finite Type . . . . .	151
<b>12</b>	<b>Role of the Critical Point</b>	<b>159</b>
12.1	The Schwarzian Derivative . . . . .	159
12.2	Critical Points and Basins of Attraction . . . . .	162
<b>13</b>	<b>Newton's Method</b>	<b>169</b>
13.1	Basic Properties . . . . .	169
13.2	Convergence and Nonconvergence . . . . .	173
<b>14</b>	<b>Fractals</b>	<b>181</b>
14.1	The Chaos Game . . . . .	181
14.2	The Cantor Set Revisited . . . . .	183
14.3	The Sierpinski Triangle . . . . .	184
14.4	The Sierpinski Carpet . . . . .	186
14.5	The Koch Snowflake . . . . .	190
14.6	Topological Dimension . . . . .	192



14.7	Fractal Dimension . . . . .	194
14.8	Iterated Function Systems . . . . .	197
14.9	Experiment: Find the Iterated Function Systems . . . . .	204
14.10	Experiment: A “Real” Chaos Game . . . . .	205
<b>15</b>	<b>Complex Functions</b>	<b>211</b>
15.1	Complex Arithmetic . . . . .	211
15.2	Complex Square Roots . . . . .	215
15.3	Linear Complex Functions . . . . .	218
15.4	Calculus of Complex Functions . . . . .	220
<b>16</b>	<b>The Julia Set</b>	<b>229</b>
16.1	The Squaring Function . . . . .	229
16.2	Another Chaotic Quadratic Function . . . . .	233
16.3	Cantor Sets Again . . . . .	235
16.4	Computing the Filled Julia Set . . . . .	240
16.5	Experiment: Filled Julia Sets and Critical Orbits . . . . .	245
16.6	The Julia Set as a Repeller . . . . .	246
<b>17</b>	<b>The Mandelbrot Set</b>	<b>251</b>
17.1	The Fundamental Dichotomy . . . . .	251
17.2	The Mandelbrot Set . . . . .	254
17.3	Complex Bifurcations . . . . .	257
17.4	Experiment: Periods of the Bulbs . . . . .	263
17.5	Experiment: Periods of the Other Bulbs . . . . .	265
17.6	Experiment: How to Add . . . . .	266
17.7	Experiment: Find the Julia Set . . . . .	267
17.8	Experiment: Similarity of the Mandelbrot Set and Julia Sets	269
<b>18</b>	<b>Other Complex Dynamical Systems</b>	<b>281</b>
18.1	Cubic Polynomials . . . . .	281
18.2	Rational Maps . . . . .	283
18.3	Exponential Functions . . . . .	291
18.4	Trigonometric Functions . . . . .	298
18.5	Complex Newton’s Method . . . . .	300
<b>A</b>	<b>Mathematical Preliminaries</b>	<b>305</b>
A.1	Functions . . . . .	305
A.2	Some Ideas from Calculus . . . . .	308
A.3	Open and Closed Sets . . . . .	309
A.4	Other Topological Concepts . . . . .	311
	<b>Bibliography</b>	<b>313</b>
	<b>Index</b>	<b>317</b>



<b>1</b>	<b>A Visual and Historical Tour</b>	<b>1</b>
1.1	Images from Dynamical Systems . . . . .	1
1.2	A Brief History of Dynamics . . . . .	4
<b>2</b>	<b>Examples of Dynamical Systems</b>	<b>17</b>
2.1	An Example from Finance . . . . .	17
2.2	An Example from Ecology . . . . .	18
2.3	Finding Roots and Solving Equations . . . . .	20
2.4	Differential Equations . . . . .	22
<b>3</b>	<b>Orbits</b>	<b>25</b>
3.1	Iteration . . . . .	25
3.2	Orbits . . . . .	26
3.3	Types of Orbits . . . . .	27
3.4	Other Orbits . . . . .	30
3.5	The Doubling Function . . . . .	31
3.6	Experiment: The Computer May Lie . . . . .	33
<b>4</b>	<b>Graphical Analysis</b>	<b>37</b>
4.1	Graphical Analysis . . . . .	37
4.2	Orbit Analysis . . . . .	39
4.3	The Phase Portrait . . . . .	41
<b>5</b>	<b>Fixed and Periodic Points</b>	<b>45</b>
5.1	A Fixed Point Theorem . . . . .	45
5.2	Attraction and Repulsion . . . . .	46
5.3	Calculus of Fixed Points . . . . .	47
5.4	Why Is This True? . . . . .	50
5.5	Periodic Points . . . . .	55
5.6	Experiment: Rates of Convergence . . . . .	57
<b>6</b>	<b>Bifurcations</b>	<b>61</b>
6.1	Dynamics of the Quadratic Map . . . . .	61
6.2	The Saddle-Node Bifurcation . . . . .	65