

Combinatorics is mathematics of enumeration, existence, construction, and optimization questions concerning finite sets. This text focuses on the first three types of questions and covers basic counting and existence principles, distributions, generating functions, recurrence relations, Pólya theory, combinatorial designs, error correcting codes, partially ordered sets, and selected applications to graph theory including the enumeration of trees, the chromatic polynomial, and introductory Ramsey theory. The only prerequisites are single-variable calculus and familiarity with sets and basic proof techniques.

The text emphasizes the brands of thinking that are characteristic of combinatorics: bijective and combinatorial proofs, recursive analysis, and counting problem classification. It is flexible enough to be used for undergraduate courses in combinatorics, second courses in discrete mathematics, introductory graduate courses in applied mathematics programs, as well as for independent study or reading courses. What makes this text a guided tour are the approximately 350 reading questions spread throughout its eight chapters. These questions provide checkpoints for learning and prepare the reader for the end-of-section exercises of which there are over 470. Most sections conclude with Travel Notes that add color to the material of the section via anecdotes, open problems, suggestions for further reading, and biographical information about mathematicians involved in the discoveries.

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