

In this primer to the many-body theory of condensed matter systems, the authors introduce the subject to the nonspecialist in a broad, concise, and up-to-date manner. A wide range of topics are covered including the second quantization of operators, coherent states, quantum-mechanical Green's functions, linear response theory, and Feynman diagrammatic perturbation theory. Material is also incorporated from quantum optics, low-dimensional systems such as graphene, and localized excitations in systems with boundaries as in nanoscale materials. More than 100 problems are included at the end of chapters, which are used both to consolidate concepts and to introduce new material. This book is suitable as a teaching tool for graduate courses and is ideal for nonspecialist students and researchers working in physics, materials science, chemistry, or applied mathematics who want to use the tools of many-body theory.

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