

Studies in Fuzziness and Soft Computing

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An Introduction to Analytical Fuzzy Plane Geometry

This book offers a rigorous mathematical analysis of fuzzy geometrical ideas. It demonstrates the use of fuzzy points for interpreting an imprecise location and for representing an imprecise line by a fuzzy line. Further, it shows that a fuzzy circle can be used to represent a circle when its description is not known precisely, and that fuzzy conic sections can be used to describe imprecise conic sections. Moreover, it discusses fundamental notions on fuzzy geometry, including the concepts of fuzzy line segment and fuzzy distance, as well as key fuzzy operations, and includes several diagrams and numerical illustrations to make the topic more understandable. The book fills an important gap in the literature, providing the first comprehensive reference guide on the fuzzy mathematics of imprecise image subsets and imprecise geometrical objects. Mainly intended for researchers active in fuzzy optimization, it also includes chapters relevant for those working on fuzzy image processing and pattern recognition. Furthermore, it is a valuable resource for beginners interested in basic operations on fuzzy numbers, and can be used in university courses on fuzzy geometry, dealing with imprecise locations, imprecise lines, imprecise circles, and imprecise conic sections.

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