

Condensed matter physics has fast become the largest discipline within physics. Based on an established course, this comprehensive textbook covers one-body, many-body, and topological perspectives. It is the first textbook that presents a comprehensive coverage of topological aspects of condensed matter as a distinct, yet integrated, component. It covers topological fundamentals and their connection to physics, introduces Berry phase and Chern numbers, describes general topological features of band structures, and delineates its classification. Applications as manifest in the quantum Hall effect, topological insulators, and Weyl semimetal are presented. Modern topics of current interest are explored in-depth, helping students prepare for cutting-edge research. These include one-electron band theory, path integrals, and coherent states functional integrals as well as Green and Matsubara functions, spontaneous symmetry breaking, superfluidity, and superconductivity. Multiple chapters covering quantum magnetism are also included. With end-of-chapter exercises throughout, it is ideal for graduate students studying advanced condensed matter physics.

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