

# AN INTRODUCTION TO INVERSE PROBLEMS IN PHYSICS

This book is a compilation of different methods of formulating and solving inverse problems in physics from classical mechanics to the potentials and nucleus-nucleus scattering. Mathematical proofs are omitted since excellent monographs already exist dealing with these aspects of the inverse problems.

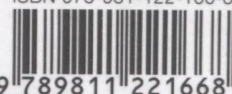
The emphasis here is on finding numerical solutions to complicated equations. A detailed discussion is presented on the use of continued fractional expansion, its power and its limitation as applied to various physical problems. In particular, the inverse problem for discrete form of the wave equation is given a detailed exposition and applied to atomic and nuclear scattering, in the latter for elastic as well as inelastic collision. This technique is also used for inverse problem of geomagnetic induction and one-dimensional electrical conductivity. Among other topics covered are the inverse problem of torsional vibration, and also a chapter on the determination of the motion of a body with reflecting surface from its reflection coefficient.

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<b>1</b>	<b>Inverse Problems in Classical Dynamics</b>	<b>5</b>
1.1	Inverse Problem for Trajectory . . . . .	5
1.2	Determination of the Shape of the Potential Energy from the Period of Oscillation . . . . .	6
1.3	Action Equivalent Hamiltonians . . . . .	8
1.4	Abel's Original Inverse Problem . . . . .	10
1.5	Inverse Scattering Problem in Classical Mechanics . . . . .	12
1.6	Inverse Problem of a Linear Chain of Masses Coupled to Springs .	15
1.7	Direct Problem of Non-exponential and of Exponential Decays in a Linear Chain . . . . .	22
1.8	Inverse Problem of Dynamics for a Non-uniform Chain . . . . .	24
1.9	Direct and Inverse Problems of Analytical Dynamics . . . . .	27
1.10	From the Classical Equations of Motion to the Lagrangian and Hamiltonian Formulations . . . . .	29
1.11	Langevin and Fokker-Planck Equations . . . . .	34
<b>2</b>	<b>Inverse Problems in Semiclassical Formulation of Quantum Mechanics</b>	<b>39</b>
2.1	Quantum Mechanical Bound States for Confining Potentials . . .	39
2.2	Semiclassical Formulation of the Inverse Scattering Problem . . .	41
<b>3</b>	<b>Inverse Problems and the Heisenberg Equations of Motion</b>	<b>47</b>
3.1	Equations of Motion Derived from the Hamiltonian Operator . . .	48
3.2	Determination of the Commutation Relations From the Equations of Motion . . . . .	49
3.3	Construction of the Hamiltonian Operator as an Inverse Problem	52
<b>4</b>	<b>Inverse Scattering Problem for the Schrödinger Equation and the Gel'fand-Levitan Formulation</b>	<b>55</b>
4.1	The Jost Solution . . . . .	56
4.2	The Jost Function . . . . .	58
4.3	The Levinson Theorem . . . . .	61

4.4	The Gel'fand-Levitan Equation . . . . .	63
4.5	Inverse Problem for One-dimensional Schrödinger Equation . . . .	68
4.6	Bargmann Potentials . . . . .	73
4.7	The Jost and Kohn Method of Inversion . . . . .	77
<b>5</b>	<b>Marchenko's Formulation of the Inverse Scattering Problem</b>	<b>83</b>
5.1	Mathematical Preliminaries . . . . .	83
5.2	Bound States Embedded in Continuum . . . . .	91
5.3	More Solvable Potentials Found from Inverse Scattering . . . . .	92
5.4	The Inverse Problem for Reflection and Transmission from a Barrier . . . . .	96
5.5	A Special Problem in Electromagnetic Inverse Scattering . . . . .	98
5.6	Construction of Reflectionless Potentials . . . . .	104
5.7	Symmetric Reflectionless Potentials Supporting a Given Set of Bound States . . . . .	108
<b>6</b>	<b>Newton-Sabatier Approach to the Inverse Problem at Fixed Energy</b>	<b>115</b>
6.1	Construction of the Potential at Fixed Energy . . . . .	115
6.2	Criticism of the Newton-Sabatier Method of Inversion at a Fixed Energy . . . . .	121
6.3	On the Results of the Numerical Solution of Inverse Problems . .	123
6.4	Modified Form of the Gel'fand-Levitan for Fixed Energy Problems and the Langer Transform . . . . .	124
6.5	Lipperheide and Fiedeldej Approach to the Inverse Problem at Fixed Energy . . . . .	130
6.6	Completeness of the Set of Jost Solutions $f(\lambda, k, r)$ . . . . .	136
6.7	Generalized Gel'fand-Levitan Approach to Inversion . . . . .	141
6.8	The Method of Schnizer and Leeb . . . . .	145
6.9	Analysis of Atom-Atom Scattering Using Complex Angular Momentum Formulation . . . . .	147
<b>7</b>	<b>Discrete Forms of the Schrödinger Equation and the Inverse Problem</b>	<b>153</b>
7.1	Zakhariev's Method . . . . .	154
7.2	The Method of Case and Kac for Discrete Form of Inverse Scattering Problem . . . . .	155
7.3	Discrete Form of the Spectral Density for Solving the Inverse Problem on Semi-axis $0 \leq r < \infty$ . . . . .	163
<b>8</b>	<b><math>\mathcal{R}</math> Matrix Theory and Inverse Problems</b>	<b>173</b>
8.1	Inverse Problem for $\mathcal{R}$ Matrix Formulation of Scattering . . . . .	178
8.2	The Finite-difference Analogue of the $\mathcal{R}$ Matrix Theory of Scattering . . . . .	179
8.3	Shell-model Hamiltonian in Tri-diagonal Form . . . . .	182
8.4	Continued Fraction Expansion of the $\mathcal{R}$ Matrix . . . . .	183

<b>9</b>	<b>Solvable Models of Fokker–Planck Equation Obtained Using the Gel’fand–Levitan Method</b>	<b>187</b>
9.1	Solution of the Fokker–Planck Equation for Symmetric and Asymmetric Double-Well Potentials . . . . .	191
<b>10</b>	<b>The Eikonal Approximation</b>	<b>195</b>
10.1	Finding the Impact Parameter Phase Shifts from the Cross Section . . . . .	201
<b>11</b>	<b>Inverse Methods Applied to Study Symmetries and Conservation Laws</b>	<b>207</b>
11.1	Classical Degeneracy and Its Quantum Counterpart . . . . .	208
11.2	Inverse Problem for Angular Momentum Eigenvalues . . . . .	209
11.3	Quantum Potentials Proportional to $\hbar$ . . . . .	213
<b>12</b>	<b>Inverse Problems in Quantum Tunneling</b>	<b>217</b>
12.1	Nonlinear Equation for Variable Reflection Amplitude . . . . .	217
12.2	Inverse One-dimensional Tunneling Problem . . . . .	219
12.3	A Method for Finding the Potential from the Reflection Amplitude . . . . .	222
12.4	Finding the Shape of the Potential Barrier in One-Dimensional Tunneling . . . . .	224
12.5	Construction of a Symmetric Double-Well Potential from the Known Energy Eigenvalues . . . . .	228
12.6	The Inverse Problem of Molecular Spectra . . . . .	230
12.7	The Inverse Problem of Tunneling for Gamow States . . . . .	233
12.8	Inverse Problem of Survival Probability . . . . .	236
<b>13</b>	<b>Inverse Problems Related to the Classical Wave Propagation</b>	<b>241</b>
13.1	Determination of the Wave Velocity in an Inhomogeneous Medium from the Reflection Coefficient . . . . .	241
13.2	Solvable Examples . . . . .	246
13.3	Extension of the Inverse Method to Reflection from a Layered Medium where the Asymptotic Values of $c(t)$ at $t \rightarrow \pm\infty$ are Different . . . . .	248
13.4	Direct and Inverse Problems of Wave Propagation Using Travel Time Coordinate . . . . .	254
13.5	$\mathcal{R}$ Matrix and the Inverse Problems of Wave Propagation . . . . .	262
13.6	Inverse Problem for Acoustic Waves: Determination of the Wave Velocity and Density Profiles . . . . .	265
13.7	Inversion of Travel Time Data in the Geometrical Acoustic Limit . . . . .	265
13.8	Riccati Equation for Solving the Direct Problem for Variable Velocity and Density . . . . .	267
13.9	Finite Difference Equation for Acoustic Pressure in an Inhomogeneous Medium: Direct and Inverse Problems . . . . .	268

13.10	Determination of the Wave Velocity and the Density of the Medium . . . . .	270
13.11	Rational Representation of the Input Data . . . . .	271
13.12	Direct and Inverse Methods Based on Continued Fraction Expansion Applied to Two Simple Models . . . . .	271
13.13	Inverse Problem of Wave Propagation Using Schwinger's Approximation . . . . .	274
<b>14</b>	<b>The Inverse Problem of Torsional Vibration</b>	<b>285</b>
<b>15</b>	<b>Local Nucleon-Nucleon Potentials Found from the Inverse Scattering Problem at Fixed Energy</b>	<b>293</b>
15.1	Constructing the $S$ Matrix from Empirical Data . . . . .	294
15.2	A Method for the Numerical Calculation of the Local Potential Using the Gel'fand-Levitan Formulation . . . . .	299
15.3	Direct and Inverse Problems for Nucleon-Nucleon Scattering Using Continued Fraction Formulation . . . . .	302
15.4	Inverse Problem of Scattering in the Presence of the Tensor Force	305
15.5	Potential Model for Generating the Input Data for Testing the Inversion Method . . . . .	309
15.6	Inverse Method of Nucleon-Nucleon Phase Shift and the Calculation of Nuclear Structure . . . . .	312
<b>16</b>	<b>The Inverse Problem of Nucleon-Nucleus Scattering</b>	<b>317</b>
16.1	Solving the Inverse Nucleon-Nucleus Problem . . . . .	320
16.2	Inverse Scattering Theory Incorporating Both Coulomb and Nuclear Forces . . . . .	324
16.3	Inverse Scattering Method for Two Identical Nuclei at Fixed Energy . . . . .	327
<b>17</b>	<b>Two Inverse Problems of Electrical Conductivity in Geophysics</b>	<b>333</b>
17.1	Inverse Problem of Electrical Conductivity in One-Dimension . .	333
17.2	The Inverse Problem of Geomagnetic Induction at a Fixed Frequency . . . . .	339
<b>18</b>	<b>Determination of the Mass Density Distribution Inside or on the Surface of a Body from the Measurement of the External Potential</b>	<b>349</b>
<b>19</b>	<b>The Inverse Problem of Reflection from a Moving Object</b>	<b>355</b>
<b>A</b>	<b>Expansion Algorithm for Continued <math>J</math>-fractions</b>	<b>361</b>
<b>B</b>	<b>Reciprocal Differences of a Quotient and Thiele's Theorem</b>	<b>367</b>
	<b>Index</b>	<b>371</b>