

Neutron scattering is arguably the most powerful technique available for looking inside materials and seeing what the atoms are doing. This textbook provides a comprehensive and up-to-date account of the many different ways neutrons are being used to investigate the behaviour of atoms and molecules in bulk matter.

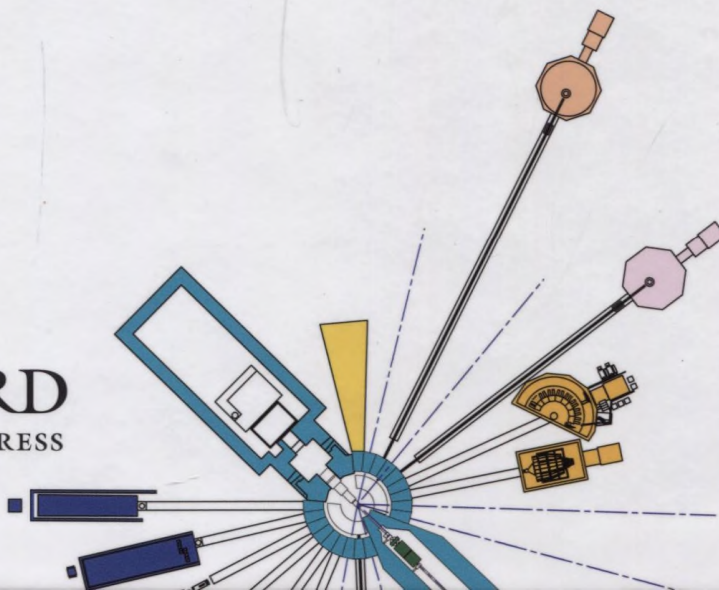
Every year, thousands of experiments are performed at neutron scattering facilities around the world, exploring phenomena in physics, chemistry, materials science, as well as in interdisciplinary areas such as biology, materials engineering, and cultural heritage. The book fulfils a need for a modern and pedagogical treatment of the principles behind the various different neutron techniques, in order to provide scientists with the essential formal tools to design their experiments and interpret the results.

The book will be of particular interest to researchers using neutrons to study the atomic-scale structure and dynamics in crystalline solids, simple liquids and molecular fluids by diffraction techniques, including small-angle scattering and reflectometry, and by spectroscopic methods, ranging from conventional techniques for inelastic and quasielastic scattering to neutron spin-echo and Compton scattering. A comprehensive treatment of magnetic neutron scattering is given, including the many and diverse applications of polarized neutrons.

- Written in a pedagogical style, including examples and exercises
- Self-contained
- Unified approach with a consistent notation, emphasizing the connections between different applications of neutron scattering
- Pays particular attention to the underlying theory behind the technique, which is important for designing experiments and interpreting the results
- Includes appendices containing relevant background and reference material
- Fully hyperlinked ebook version

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