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Dynamics of Viscous Compressible Fluids
Eduard Feireisl

This book develops the most recent ideas and concepts of the mathematical theory of viscous, compressible and heat conducting fluids. Two main goals are pursued: (i) global existence theory within the framework of variational (weak) solutions for the full system of the Navier-Stokes equations supplemented with large data; and (ii) optimal existence results for the barotropic flows with respect to the available *a priori* estimates.

The book is intended to be a compact and self-contained presentation of the most recent results of the mathematical theory of viscous compressible fluids. In order to place the text in better perspective, each chapter is concluded with a section devoted to historical notes including references to all important and new results. The material is by no means intended to be the last word on the subject but rather to indicate possible directions of future research. It is aimed at research mathematicians, theoretical physicists, engineers and graduate students.

Eduard Feireisl is a researcher at the Mathematical Institute of the Czech Academy of Sciences, Prague.

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