

MATHEMATICAL STRUCTURES *of* QUANTUM MECHANICS

This marvelous book aims to strengthen the mathematical background and sharpen the tools of mathematics for students without rigorous training before taking the quantum mechanics course. The abstract construction of quantum postulates in the framework of Hilbert space and Hermitian operators is realized by q -representation in the formulation to demonstrate the conventional approach to quantum theory.

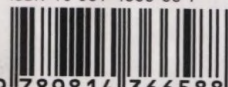
Symmetry property is emphasized and is extensively explored in this book both in continuous transformations as well as discrete ones. The space-time structure is discussed in depth and the Dirac equation is formulated by symmetry consideration of the Lorentz group.



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