

Contents

Preface	xiii
1 Probability and counting	1
1.1 Why study probability?	1
1.2 Sample spaces and Pebble World	3
1.3 Naive definition of probability	6
1.4 How to count	8
1.5 Story proofs	20
1.6 Non-naive definition of probability	21
1.7 Recap	26
1.8 R	29
1.9 Exercises	33
2 Conditional probability	45
2.1 The importance of thinking conditionally	45
2.2 Definition and intuition	46
2.3 Bayes' rule and the law of total probability	52
2.4 Conditional probabilities are probabilities	59
2.5 Independence of events	63
2.6 Coherency of Bayes' rule	67
2.7 Conditioning as a problem-solving tool	68
2.8 Pitfalls and paradoxes	74
2.9 Recap	79
2.10 R	80
2.11 Exercises	83
3 Random variables and their distributions	103
3.1 Random variables	103
3.2 Distributions and probability mass functions	106
3.3 Bernoulli and Binomial	112
3.4 Hypergeometric	115
3.5 Discrete Uniform	118
3.6 Cumulative distribution functions	120
3.7 Functions of random variables	123
3.8 Independence of r.v.s	129
3.9 Connections between Binomial and Hypergeometric	133
3.10 Recap	136

3.11	R	138
3.12	Exercises	140
4	Expectation	149
4.1	Definition of expectation	149
4.2	Linearity of expectation	152
4.3	Geometric and Negative Binomial	157
4.4	Indicator r.v.s and the fundamental bridge	164
4.5	Law of the unconscious statistician (LOTUS)	170
4.6	Variance	171
4.7	Poisson	174
4.8	Connections between Poisson and Binomial	181
4.9	*Using probability and expectation to prove existence	184
4.10	Recap	189
4.11	R	192
4.12	Exercises	194
5	Continuous random variables	213
5.1	Probability density functions	213
5.2	Uniform	220
5.3	Universality of the Uniform	224
5.4	Normal	231
5.5	Exponential	238
5.6	Poisson processes	244
5.7	Symmetry of i.i.d. continuous r.v.s	248
5.8	Recap	250
5.9	R	253
5.10	Exercises	255
6	Moments	267
6.1	Summaries of a distribution	267
6.2	Interpreting moments	272
6.3	Sample moments	276
6.4	Moment generating functions	279
6.5	Generating moments with MGFs	283
6.6	Sums of independent r.v.s via MGFs	286
6.7	*Probability generating functions	287
6.8	Recap	292
6.9	R	293
6.10	Exercises	298
7	Joint distributions	303
7.1	Joint, marginal, and conditional	304
7.2	2D LOTUS	324
7.3	Covariance and correlation	326

7.4	Multinomial	332
7.5	Multivariate Normal	337
7.6	Recap	343
7.7	R	346
7.8	Exercises	348
8	Transformations	367
8.1	Change of variables	369
8.2	Convolutions	375
8.3	Beta	379
8.4	Gamma	387
8.5	Beta-Gamma connections	396
8.6	Order statistics	398
8.7	Recap	402
8.8	R	404
8.9	Exercises	407
9	Conditional expectation	415
9.1	Conditional expectation given an event	415
9.2	Conditional expectation given an r.v.	424
9.3	Properties of conditional expectation	426
9.4	*Geometric interpretation of conditional expectation	431
9.5	Conditional variance	432
9.6	Adam and Eve examples	436
9.7	Recap	439
9.8	R	441
9.9	Exercises	443
10	Inequalities and limit theorems	457
10.1	Inequalities	458
10.2	Law of large numbers	467
10.3	Central limit theorem	471
10.4	Chi-Square and Student- t	477
10.5	Recap	480
10.6	R	483
10.7	Exercises	486
11	Markov chains	497
11.1	Markov property and transition matrix	497
11.2	Classification of states	502
11.3	Stationary distribution	506
11.4	Reversibility	513
11.5	Recap	520
11.6	R	521
11.7	Exercises	524

12 Markov chain Monte Carlo	535
12.1 Metropolis-Hastings	536
12.2 Gibbs sampling	548
12.3 Recap	554
12.4 R	555
12.5 Exercises	557
13 Poisson processes	559
13.1 Poisson processes in one dimension	559
13.2 Conditioning, superposition, and thinning	561
13.3 Poisson processes in multiple dimensions	573
13.4 Recap	575
13.5 R	575
13.6 Exercises	577
A Math	581
A.1 Sets	581
A.2 Functions	585
A.3 Matrices	590
A.4 Difference equations	592
A.5 Differential equations	593
A.6 Partial derivatives	594
A.7 Multiple integrals	594
A.8 Sums	596
A.9 Pattern recognition	599
A.10 Common sense and checking answers	599
B R	601
B.1 Vectors	601
B.2 Matrices	602
B.3 Math	602
B.4 Sampling and simulation	603
B.5 Plotting	603
B.6 Programming	603
B.7 Summary statistics	604
B.8 Distributions	604
C Table of distributions	605
References	607
Index	609