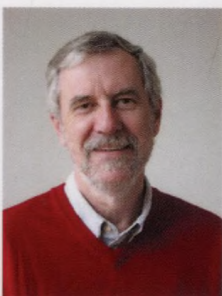


Algorithms are a dominant force in modern culture, and every indication is that they will become more pervasive, not less. The best algorithms are undergirded by beautiful mathematics. This text cuts across discipline boundaries to highlight some of the most famous and successful algorithms. Readers are exposed to the principles behind these examples and guided in assembling complex algorithms from simpler building blocks.

Algorithms from THE BOOK

- incorporates Julia code for easy experimentation,
- is written in clear, concise prose consistent with mathematical rigor,
- includes a large number of classroom-tested exercises at the end of each chapter, and
- covers background material, often omitted from undergraduate courses, in the appendices.

This textbook is aimed at first-year graduate and advanced undergraduate students. It will also serve as a convenient reference for professionals throughout the mathematical sciences, physical sciences, engineering, and the quantitative sectors of the biological and social sciences.



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