

# Contents

Preface	xviii
<b>I CONCEPTUAL BASES OF EXPERIMENTAL DESIGN AND ANALYSIS</b>	
<b>1 The Logic of Experimental Design and Analysis</b>	<b>3</b>
Overview of Chapter: Research Questions Addressed	3
Published Example	3
Philosophy of Science	4
The Traditional View of Science	4
Responses to the Criticisms of the Idea of Pure Science	6
Assumptions	6
Modern Philosophy of Science	11
Introduction to the Fisher Tradition	24
“Interpretation and Its Reasoned Basis”	25
A Discrete Probability Example	26
Randomization Test	31
Of Hypotheses and $p$ Values: Fisher Versus Neyman-Pearson	37
Toward Tests Based on Distributional Assumptions	40
Statistical Tests With Convenience Samples	40
The Assumption of Normality	41
Summary of Main Points	47
Important Formulas	47
Online Materials Available on <i>DesigningExperiments.com</i>	48
Exercises	48
<b>2 Drawing Valid Inferences From Experiments</b>	<b>59</b>
Overview of Chapter: Research Questions Addressed	59
Published Example	59
Threats to the Validity of Inferences From Experiments	60
Types of Validity	61
Statistical Conclusion Validity	62
Internal Validity	64
Construct Validity	66
External Validity	68
Conceptualizing and Controlling for Threats to Validity	69



Overview of Experimental Designs to Be Considered	71
Summary of Main Points	75
Exercises	76
<b>II MODEL COMPARISONS FOR BETWEEN-SUBJECTS DESIGNS</b>	
<b>3 Introduction to Model Comparisons: One-Way Between-Subjects Designs</b>	<b>83</b>
Overview of Chapter: Research Questions Addressed	83
Published Example	83
Introduction	84
The General Linear Model	86
One-Group Situation	88
Basics of Models	88
Optional	90
Proof That $\bar{Y}$ Is the Least-Squares Estimate of $\mu$	90
Development of the General Form of the Test Statistic	92
Numerical Example	94
Relationship of Models and Hypotheses	97
Two-Group Situation	97
Development in Terms of Models	97
Alternative Development and Identification With Traditional Terminology	100
The General Case of One-Way Designs	102
Formulation in Terms of Models	102
Numerical Example	106
A Model in Terms of Effects	108
Parameter Estimates	110
Computation of the Test Statistic	111
On Tests of Significance and Measures of Effect	112
Measures of Effect	114
Measures of Effect Size	116
Mean Difference	116
Confidence Intervals	116
Estimated Effect Parameters	119
The Standardized Difference Between Means	120
Confidence Intervals for Standardized Differences Between Means	122
Standardized Effects, and the Signal-to-Noise Ratio	126
Measures of Association Strength	127
Confidence Intervals for Measures of Association Strength	130
Evaluation of Measures	130
Alternative Representations of Effects	132
Binomial Effect Size Display (BESD)	132
Common Language (CL) Effect Size	132
Graphical Methods	133



Statistical Assumptions	133
Implications for Expected Values	134
Robustness of ANOVA	135
Checking for Normality and Homogeneity of Variance	138
Transformations	141
Power of the $F$ Test: One-Way ANOVA	144
Determining an Appropriate Sample Size	145
Specifying the Minimally Important Difference	146
Specifying Population Parameters and Using Power Charts	146
Determining Sample Size Using $\delta$ and Table 3.10	148
Pilot Data and Observed Power	149
Summary of Main Points	152
Important Formulas	153
Online Materials Available on <i>DesigningExperiments.com</i>	155
Exercises	156
<b>4 Individual Comparisons of Means</b>	<b>170</b>
Overview of Chapter: Research Questions Addressed	170
Published Example	170
Introduction	171
A Model Comparison Approach for Testing Individual Comparisons	172
Preview of Individual Comparisons	172
Relationship to Model Comparisons	172
Expression of $F$ Statistic	174
Numerical Example	176
Complex Comparisons	177
Models Perspective	177
Numerical Example	183
The $t$ Test Formulation of Hypothesis Testing for Contrasts	185
Practical Implications	185
Unequal Population Variances	187
Numerical Example	190
Practical Implications	191
Measures of Effect	191
Measures of Effect Size	192
Confidence Intervals	192
Standardized Difference	194
Measures of Association Strength	195
Testing More Than One Contrast	199
How Many Contrasts Should Be Tested?	199
Linear Independence of Contrasts	200
Orthogonality of Contrasts	201
Summary of Main Points	203
Important Formulas	203
Online Materials Available on <i>DesigningExperiments.com</i>	204
Exercises	204



<b>5 Testing Several Contrasts: The Multiple-Comparisons Problem</b>	<b>216</b>
Overview of Chapter: Research Questions Addressed	216
Published Example	217
Multiple Comparisons	217
Experimentwise and Per-Comparison Error Rates	217
Simultaneous Confidence Intervals	220
Levels of Strength of Inference	221
Types of Contrasts	222
Overview of Techniques	223
Planned Versus Post Hoc Contrasts	224
Multiple Planned Comparisons	225
Bonferroni Adjustment	226
Modification of the Bonferroni Approach With Unequal Variances	229
Numerical Example	230
Pairwise Comparisons	233
Tukey's HSD Procedure	234
Modifications of Tukey's HSD	236
Numerical Example	238
Post Hoc Complex Comparisons	239
Proof That $SS_{\max} = SS_B$	240
Comparison of Scheffé to Bonferroni and Tukey	242
Modifications of Scheffé's Method	244
Numerical Example	245
Other Multiple-Comparison Procedures	247
Dunnett's Procedure for Comparisons With a Control	247
Numerical Example	247
Procedures for Comparisons With the Best	248
Numerical Example	252
Fisher's LSD (Protected $t$ )	254
False Discovery Rate	256
Choosing an Appropriate Procedure	260
Summary of Main Points	263
Important Formulas	264
Online Materials Available at <i>DesigningExperiments.com</i>	265
Exercises	265
<b>6 Trend Analysis</b>	<b>275</b>
Overview of Chapter: Research Questions Addressed	275
Published Example	276
Quantitative Factors	276
Statistical Treatment of Trend Analysis	277
The Slope Parameter	278
Numerical Example	280
Hypothesis Test of Slope Parameter	282
Confidence Interval and Other Effect Size Measures for the Slope Parameter	284
Numerical Example	284



Testing for Nonlinearity	286
Numerical Example	289
Testing Individual Higher Order Trends	290
Contrast Coefficients for Higher Order Trends	291
Numerical Example	293
Further Examination of Nonlinear Trends	296
Trend Analysis With Unequal Sample Sizes	300
Concluding Comments	301
Summary of Main Points	302
Important Formulas	302
Online Materials Available on <i>DesigningExperiments.com</i>	303
Exercises	303
<b>7 Two-Way Between-Subjects Factorial Designs</b>	<b>312</b>
Overview of Chapter: Research Questions Addressed	312
Published Example	313
Introduction	313
The $2 \times 2$ Design	313
The Concept of Interaction	315
Additional Perspectives on the Interaction	316
A Model Comparison Approach to the General Two-Factor Design	318
Alternate Form of Full Model	319
Comparison of Models for Hypothesis Testing	322
Numerical Example	328
Familywise Control of Alpha Level	329
Measures of Effect	329
Follow-Up Tests	335
Further Investigation of Main Effects	335
Further Investigation of an Interaction—Simple Effects	337
Relationships of Main Effect, Interaction, and Simple Effects	341
Consideration of Type I Error Rate in Testing Simple Effects	343
Error Term for Testing Simple Effects	345
An Alternative Method for Investigating an Interaction—Interaction Contrasts	345
Statistical Power	354
Advantages of Factorial Designs	355
Nonorthogonal Designs	356
Design Considerations	357
Relationship Between Design and Analysis	358
Analysis of the $2 \times 2$ Nonorthogonal Design	358
Test of the Interaction	359
Unweighted Marginal Means and Type III Sum of Squares	361
Unweighted Versus Weighted Marginal Means	362
Type II Sum of Squares	363
Summary of Three Types of Sum of Squares	364
Analysis of the General $a \times b$ Nonorthogonal Design	365
Test of the Interaction	366



Test of Unweighted Marginal Means	366
Test of Marginal Means in an Additive Model	368
Test of Weighted Marginal Means	369
Summary of Types of Sum of Squares	370
Which Type of Sum of Squares Is Best?	370
A Note on Statistical Software for Analyzing Nonorthogonal Designs	372
Numerical Example	374
Final Remarks	379
Summary of Main Points	379
Important Formulas	379
Online Materials Available on <i>DesigningExperiments.com</i>	382
Exercises	382
<b>8 Higher-Order Between-Subjects Factorial Designs</b>	<b>401</b>
Overview of Chapter: Research Questions Addressed	401
Published Example	401
The $2 \times 2 \times 2$ Design	402
The Meaning of Main Effects	403
The Meaning of Two-Way Interactions	404
The Meaning of the Three-Way Interaction	405
Graphical Depiction	407
Further Consideration of the Three-Way Interaction	409
Summary of Meaning of Effects	413
The General $A \times B \times C$ Design	414
The Full Model	414
Formulation of Restricted Models	415
Numerical Example	419
Implications of a Three-Way Interaction	422
General Guideline for Analyzing Effects	423
Summary of Results	429
Graphical Depiction of Data	430
Confidence Intervals for Single Degree of Freedom Effects	431
Other Questions of Potential Interest	434
Tests to Be Performed When the Three-Way Interaction Is Non-Significant	435
Nonorthogonal Designs	437
Higher Order Designs	439
Summary of Main Points	440
Important Formulas	441
Online Materials Available on <i>DesigningExperiments.com</i>	441
Exercises	441
<b>9 Designs With Covariates: ANCOVA and Blocking</b>	<b>451</b>
Overview of Chapter: Research Questions Addressed	451
Published Example	451
Introduction	452
ANCOVA	454



The Logic of ANCOVA	454
Linear Models for ANCOVA	455
Parameter Estimates	456
Comparison of Models	465
Two Consequences of Using ANCOVA	467
Test of Regression	467
Estimated Conditional Means	468
Examples of Adjusted Effects	471
Summary	473
Assumptions in ANCOVA	473
Basic Implications	474
Lack of Independence of Treatment and Covariate	475
Summary Regarding Lack of Independence of Treatment and Covariate	481
Measurement Error in Covariate	481
Numerical Example	483
Measures of Effect	486
Comparisons Among Adjusted Group Means	489
Generalizations of the ANCOVA Model	492
Multiple Covariates	492
Nonlinear Relationships	493
Multifactor Studies	493
Choosing Covariates in Randomized Designs	494
Sample Size Planning and Power Analysis in ANCOVA	495
Alternate Methods of Analyzing Designs With Concomitant Variables	498
ANOVA of Residuals	498
Gain Scores	498
Blocking	502
Conclusions Regarding Blocking	507
Matching: Propensity Scores	507
Summary of Main Points	510
Important Formulas	510
Online Materials Available on <i>DesigningExperiments.com</i>	511
Exercises	512
Extension: Heterogeneity of Regression	518
Test for Heterogeneity of Regression	518
Accommodating Heterogeneity of Regression	523
Simultaneous Tests	530
Carrying Out Tests and Determining Regions of Significance	531
Summary Regarding Heterogeneity of Regression	536
Important Formulas	537
Exercises	538
<b>10 Designs With Random or Nested Factors</b>	<b>547</b>
Overview of Chapter: Research Questions Addressed	547
Published Example	547



Designs With Random Factors	548
Introduction to Random Effects	548
One-Factor Case	550
Model	550
Model Comparisons	552
Expected Values	552
Two-Factor Case	553
Expected Mean Squares	553
Model Comparisons	556
Selection of Error Terms	558
Numerical Example	560
Alternative Tests and Design Considerations With Random Factors	562
Follow-Up Tests and Confidence Intervals	563
Measures of Association Strength	564
Intraclass Correlation	565
Numerical Example	566
Using Statistical Computer Programs to Analyze Designs With Random Factors	568
Determining Power in Designs With Random Factors	569
Designs With Nested Factors	572
Introduction to Nested Factors	572
Example	578
Models and Tests	578
Degrees of Freedom	584
Statistical Assumptions and Related Issues	585
Follow-Up Tests and Confidence Intervals	586
Standardized Effect Size Estimates	587
Strength of Association in Nested Designs	588
Using Statistical Computer Programs to Analyze Nested Designs	590
Selection of Error Terms When Nested Factors Are Present	591
Complications That Arise in More Complex Designs	593
Summary of Main Points	597
Important Formulas	598
Online Materials Available on <i>DesigningExperiments.com</i>	601
Exercises	601

### III MODEL COMPARISONS FOR DESIGNS INVOLVING WITHIN-SUBJECTS FACTORS

<b>11 One-Way Within-Subjects Designs: Univariate Approach</b>	<b>611</b>
Overview of Chapter: Research Questions Addressed	611
Published Example	611
Prototypical Within-Subjects Designs	612
Advantages of Within-Subjects Designs	613
Analysis of Repeated-Measures Designs With Two Levels	614



The Problem of Correlated Errors	614
Reformulation of Model	616
Analysis of Within-Subjects Designs With More Than Two Levels	618
Traditional Univariate (Mixed-Model) Approach	619
Comparison of Full and Restricted Models	620
Estimation of Parameters: Numerical Example	621
Assumptions in the Traditional Univariate (Mixed-Model) Approach	627
Homogeneity, Sphericity, and Compound Symmetry	627
Numerical Example	628
Adjusted Univariate Tests	630
Lower-Bound Adjustment	630
$\hat{\epsilon}$ Adjustment	631
$\tilde{\epsilon}$ Adjustment	632
Summary of Four Mixed-Model Approaches	632
Measures of Effect	634
Comparisons Among Individual Means	637
Confidence Intervals for Comparisons	638
Optional	640
Confidence Intervals With Pooled and Separate Variances	640
Considerations in Designing Within-Subjects Experiments	643
Order Effects	643
Differential Carryover Effects	644
Controlling for Order Effects With More Than Two Levels: Latin Square Designs	645
Relative Advantages of Between-Subjects and Within-Subjects Designs	649
Intraclass Correlations for Assessing Reliability	652
Summary of Main Points	655
Important Formulas	656
Online Materials Available on <i>DesigningExperiments.com</i>	658
Exercises	658
<b>12 Higher-Order Designs With Within-Subjects Factors:</b>	
<b>Univariate Approach</b>	<b>668</b>
Overview of Chapter: Research Questions Addressed	668
Published Example	668
Designs With Two Within-Subjects Factors	669
Omnibus Tests	669
Numerical Example	673
Further Investigation of Main Effects	674
Further Investigation of an Interaction—Simple Effects	676
Interaction Contrasts	678
Statistical Packages and Pooled Error Terms Versus Separate Error Terms	679
Assumptions	679
Adjusted Univariate Tests	684
Confidence Intervals	686
Quasi- $F$ Ratios	686



One Within-Subjects Factor and One Between-Subjects Factor in the Same Design	688
Omnibus Tests	690
An Appropriate Full Model	690
Restricted Models	691
Error Terms	692
Numerical Example	694
Further Investigation of Main Effects	695
Between-Subjects Factor	695
Within-Subjects Factor	695
Further Investigation of an Interaction—Simple Effects	697
Within-Subjects Effects at a Fixed Level of Between-Subjects Factor	697
Between-Subjects Effects at a Fixed Level of Within-Subjects Factor	699
Interaction Contrasts	701
Assumptions	704
Adjusted Univariate Tests	706
More Complex Designs	706
Designs With Additional Factors	706
Latin Square Designs	707
Summary of Main Points	712
Important Formulas	712
Online Materials Available on <i>DesigningExperiments.com</i>	714
Exercises	714
<b>13 One-Way Within-Subjects Designs: Multivariate Approach</b>	<b>728</b>
Overview of Chapter: Research Questions Addressed	728
Published Example	728
A Brief Review of Analysis for Designs With Two Levels	729
Multivariate Analysis of Within-Subjects Designs With Three Levels	730
Need for Multiple $D$ Variables	731
Full and Restricted Models	732
The Relationship Between $D_1$ and $D_2$	734
Matrix Formulation and Determinants	735
Test Statistic	740
Multivariate Analysis of Within-Subjects Designs With $a$ Levels	741
Forming $D$ Variables	741
Test Statistic	742
Numerical Example	742
Measures of Effect	745
Choosing an Appropriate Sample Size	746
Choice of $D$ Variables	753
Tests of Individual Contrasts	755
Multiple-Comparison Procedures: Determination of Critical Values	757
Planned Comparisons	757
Pairwise Comparisons	757
Post Hoc Complex Comparisons	758
Confidence Intervals for Contrasts	759



The Relationship Between the Multivariate Approach and the Mixed-Model Approach	762
Orthonormal Contrasts	763
Comparison of the Two Approaches	765
Multivariate and Mixed-Model Approaches for Testing Contrasts	767
Numerical Example	768
The Difference in Error Terms	770
Which Error Term Is Better?	771
A General Comparison of the Multivariate and Mixed-Model Approaches	773
Assumptions	774
Tests of Contrasts	774
Type I Error Rates	775
Type II Error Rates	775
Summary	777
Summary of Main Points	779
Important Formulas	779
Online Materials Available on <i>DesigningExperiments.com</i>	780
Exercises	781
<b>14 Higher-Order Designs With Within-Subjects Factors:</b>	
<b>Multivariate Approach</b>	<b>790</b>
Overview of Chapter: Research Questions Addressed	790
Published Example	790
Two Within-Subjects Factors, Each With Two Levels	791
Formation of Main Effect <i>D</i> Variables	792
Formation of Interaction <i>D</i> Variables	795
Relationship to the Mixed-Model Approach	796
Multivariate Analysis of Two-Way $a \times b$ Within-Subjects Designs	797
Formation of Main Effect <i>D</i> Variables	797
Formation of Interaction <i>D</i> Variables	799
Omnibus Tests—Multivariate Significance Tests	802
Measures of Effect	803
Further Investigation of Main Effects	804
Further Investigation of an Interaction—Simple Effects	805
Interaction Contrasts	807
Confidence Intervals for Contrasts	808
Multivariate and Mixed-Model Approaches for Testing Contrasts	810
Comparison of the Multivariate and Mixed-Model Approaches	811
One Within-Subjects Factor and One Between-Subjects Factor in the Same Design	811
Split-Plot Design With Two Levels of the Within-Subjects Factor	811
Main Effect of Between-Subjects Factor	812
Within-Subjects Effects	814
Test of the Interaction	816
Within-Subjects Main Effect	816
Summary	819



General $a \times b$ Split-Plot Design	820
Between-Subjects Main Effect	821
Within-Subjects Effects	822
Within-Subjects Main Effect	823
Test of the Interaction	826
Measures of Effect	833
Further Investigation of Main Effects	833
Further Investigation of an Interaction—Simple Effects	836
Between-Subjects Effects at a Fixed Level of Within-Subjects Factor	836
Within-Subjects Effects at a Fixed Level of Between-Subjects Factor	837
Cell Mean Comparisons	840
Interaction Contrasts	842
Confidence Intervals for Contrasts	844
Assumptions of the Multivariate Approach	848
Multivariate and Mixed-Model Approaches for Testing Within-Subjects Contrasts	849
Comparison of the Multivariate and Mixed-Model Approaches	850
Optional	850
More Complex Designs	850
Summary of Main Points	856
Important Formulas	857
Two-Way Within-Subjects Designs	857
Split-Plot Designs	857
Online Materials Available on <i>DesigningExperiments.com</i>	858
Exercises	859

## IV MIXED-EFFECTS MODELS

### 15 An Introduction to Mixed-Effects Models:

<b>Within-Subjects Designs</b>	<b>877</b>
Overview of Chapter: Research Questions Addressed	877
Published Example	878
Introduction	878
Advantages of Mixed-Effects Models	879
Within-Subjects Designs	879
Overview of Remainder of Chapter	880
Within-Subjects Designs	880
Various Types of Within-Subjects Designs	880
Models for Longitudinal Data	881
Review of the ANOVA Mixed-Model Approach	881
Mixed-Effects Models	883
A Maximum Likelihood Approach	883
An Example of Maximum Likelihood Estimation	883
Comparison of ANOVA and Maximum Likelihood Models	886
Numerical Example	889
A Closer Look at the Random Effects Model	894



Graphical Representation of Longitudinal Data	895
Graphical Representation of the Random Intercept Model	897
Coding Random Effects Predictor Variables	901
Random Effects Parameters	902
Numerical Example	904
Graphical Representation of a Model With Random Slope and Intercept	906
Further Consideration of Competing Models	907
Additional Models	909
Straight-Line Change Model	912
Graphical Representation of a Growth Curve Model	915
Design Considerations	917
An Alternative Approach and Conceptualization	918
Additional Covariance Matrix Structures	926
Tests of Contrasts	930
Overview of Broader Model Comparison	931
Complex Designs	933
Factorial Fixed Effects	933
Multiple Variables Measured Over Time	934
Unbalanced Designs	935
Summary of Main Points	937
Important Formulas	937
Online Materials Available on <i>DesigningExperiments.com</i>	937
Exercises	938
<b>16 An Introduction to Mixed-Effect Models: Nested Designs</b>	<b>950</b>
Overview of Chapter: Research Questions Addressed	950
Published Example	951
Introduction	951
Review of the ANOVA Approach	952
Mixed-Effects Models Analysis for the Simple Nested Design	954
Numerical Example—Equal $n$	956
Numerical Example—Unequal $n$	964
Mixed-Effects Models for Complex Nested Designs	969
Hierarchical Representation of the Model for a Simple Nested Design	971
Models With Additional Level 2 Variables	973
Models With Additional Level 1 Variables	977
Summary of Main Points	991
Important Formulas	991
Online Materials Available on <i>DesigningExperiments.com</i>	992
Exercises	992
Appendix	998
References	1026
Name Index	1041
Subject Index	1049