

Oscillation Theory of Two-Term Differential Equations

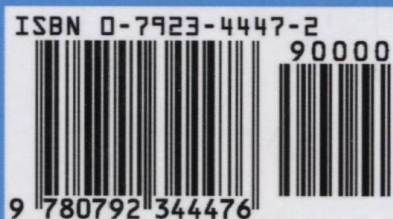
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This volume is about oscillation theory. In particular, it considers the two-term linear differential equations $L_n y + p(x)y = 0$, where L_n is a disconjugate operator of order n and $p(x)$ has a fixed sign. Special attention is paid to the equation $y^{(n)} + p(x)y = 0$. These equations enjoy a very rich structure and are the natural generalization of the Sturm–Liouville operator. Our aim is to introduce an order among the results which are distributed over hundreds of research papers, and arrange them in a unified and self-contained way. Many new proofs are given and the original proof is never copied verbatim. Numerous new results are included. Among the topics which are discussed are oscillation and nonoscillation, disconjugacy, various types of disfocality, extremal configurations of zeros, comparison theorems, classification of solutions according to their behaviour near infinity and their dominance properties.

Audience

This work will be of interest to researchers and graduate students interested in the qualitative theory of differential equations.



PREFACE	vii
0. INTRODUCTION	1
0.1 Linear differential equations and disconjugacy	1
0.2 Pólya's factorization	3
0.3 Willet's notation	7
0.4 Green's function	8
1. THE BASIC LEMMA	11
1.1 Two term equations	11
1.2 The basic lemma	12
1.3 Examples	21
2. BOUNDARY VALUE FUNCTIONS	27
2.1 Boundary value functions	27
2.2 More boundary value problems	31
2.3 Extremal points and boundary value problems	32
3. BASES OF SOLUTIONS	42
3.1 A base of solutions	42
3.2 Adjoint equations and boundary value problems	45
4. COMPARISON OF BOUNDARY VALUE PROBLEMS	49
4.1 Ordering of boundary value problems	49
4.2 m -poised boundary value problems	55
4.3 Comparison of Green's functions	57
5. COMPARISON THEOREMS FOR TWO EQUATIONS	60
5.1 Pointwise comparison of coefficients	60
5.2 Integral comparison of coefficients	65
6. DISFOCALITY AND ITS CHARACTERIZATION	67
6.1 Characterization of disfocality	67
6.2 Integral characterization of disfocality	73
6.3 A lemma of Kiguradze	78
6.4 Explicit integral criteria for disfocality	81
6.5 Asymptotic behaviour of solutions	87
6.6 Euler's equation and disfocality	91
6.7 Green's function of the focal boundary value problem	95
7. VARIOUS TYPES OF DISFOCALITY	98
7.1 Comparison of various types of disfocality	98
7.2 Application to the equation $y^{(n)} + py = 0$	105
7.3 Comparison theorems with integrated coefficients	107
8. SOLUTIONS ON AN INFINITE INTERVAL	111
8.1 A classification of the solutions	111
8.2 Nonoscillation and eventual disconjugacy	120
8.3 Equivalence of disconjugacy and disfocality	121

8.4 The limit of Green's functions	124
8.5 Property A, Property B and strong oscillation	126
8.6 Nonoscillation	129
9. DISCONJUGACY AND ITS CHARACTERIZATION	131
9.1 Disconjugacy and disfocality	131
9.2 Characterization of $(k, n - k)$ -disconjugacy	133
9.3 Explicit criteria for disconjugacy	136
9.4 A transformation of the equation	139
10. EIGENVALUE PROBLEMS	141
10.1 Existence of eigenvalues	141
10.2 Zeros of eigenfunctions	146
10.3 Dependence on the boundary conditions	152
10.4 Comparison of two-point boundary value problems	156
11. MORE EXTREMAL POINTS	161
11.1 The i th conjugate point	161
11.2 The focal point	164
11.3 Maximal gap between zeros	166
12. MINORS OF THE WRONSKIAN	176
12.1 Minors as solutions of a differential equation	176
12.2 Zeros of odd order minors	185
12.3 Applications to oscillation	191
13. THE DOMINANCE PROPERTY OF SOLUTIONS	193
13.1 Dominance	193
13.2 Comparison of minors	200
13.3 Dichotomy of a nonoscillatory set	205
13. REFERENCES	209
13. INDEX	217