Variational and Non-variational Methods in Nonlinear Analysis and Boundary Value Problems

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This book provides a comprehensive exposition of modern topics in nonlinear analysis with applications to various boundary value problems with discontinuous nonlinearities and nonsmooth constraints. Our framework includes multivalued elliptic problems with discontinuities, variational inequalities, hemivariational inequalities and evolution problems. In addition to the existence of solutions, a major part of the book is devoted to the study of different qualitative properties such as multiplicity, location, extremality, and stability. The treatment relies on variational methods, monotonicity principles, topological arguments and optimization techniques. The book is based on the authors' original results obtained in the last decade. A great deal of the material is published for the first time in this book and is organized in a unified way. The book is self-contained. The abstract results are illustrated through various examples and applications.

Audience

Faculty, graduate students, pure and applied mathematicians, researchers in mechanics, physics, and engineering.

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