

Order Structure and Topological Methods in Nonlinear Partial Differential Equations

The maximum principle induces an order structure for partial differential equations, and has become an important tool in nonlinear analysis. This book is the first of two volumes to systematically introduce the applications of order structure in certain nonlinear partial differential equation problems.

The maximum principle is revisited through the use of the Krein–Rutman theorem and the principal eigenvalues. Its various versions, such as the moving plane and sliding plane methods, are applied to a variety of important problems of current interest. The upper and lower solution method, especially its weak version, is presented in its most up-to-date form with enough generality to cater for wide applications. Recent progress on the boundary blow-up problems and their applications are discussed, as well as some new symmetry and Liouville type results over half and entire spaces. Some of the results included here are published for the first time.

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