

This wide-ranging and self-contained account of the spectral theory of non-self-adjoint linear operators is ideal for postgraduate students and researchers, and contains many illustrative examples and exercises.

Fredholm theory, Hilbert–Schmidt and trace class operators are discussed, as are one-parameter semigroups and perturbations of their generators. Two chapters are devoted to using these tools to analyze Markov semigroups.

The text also provides a thorough account of the new theory of pseudospectra, and presents the recent analysis by the author and Barry Simon of the form of the pseudospectra at the boundary of the numerical range. This was a key ingredient in the determination of properties of the zeros of certain orthogonal polynomials on the unit circle.

Finally, two methods, both very recent, for obtaining bounds on the eigenvalues of non-self-adjoint Schrödinger operators are described. The text concludes with a description of the surprising spectral properties of the non-self-adjoint harmonic oscillator.

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