

Measure theory is a classical area of mathematics that continues to enjoy intensive development and has fruitful connections with most other fields of mathematics as well as important applications in physics.

This book gives a systematic presentation of modern measure theory as it has developed over the past century and offers three levels of presentation: a standard university graduate course, an advanced study containing some complements to the basic course (the material of this level corresponds to a variety of special courses), and, finally, more specialized topics partly covered by more than 850 exercises. Bibliographical and historical comments and an extensive bibliography with 2000 works covering more than a century are provided.

Volume 1 is devoted to the classical theory of measure and integral. Whereas this represents the ideas that go back mainly to Lebesgue, the second volume is to a large extent the result of the later development up to recent years. The central subjects of Volume 2 are: transformations of measures, conditional measures, and weak convergence of measures. These topics are closely interwoven and form the heart of modern measure theory.

The target readership includes graduate students seeking to acquire deeper knowledge of measure theory, instructors of courses in measure and integration theory, and researchers in all fields of mathematics. The book may also serve as a reference for physicists and other scientists.

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