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A complete introduction to partial differential equations, this textbook provides a rigorous yet accessible guide to students in mathematics, physics, and engineering. The presentation is lively and up to date, putting particular emphasis into developing an appreciation of underlying mathematical theory.

Beginning with basic definitions, properties, and derivations of some fundamental equations from mathematical physics, the book studies first-order equations, classification of second-order equations, and the one-dimensional wave equation. Two chapters are devoted to the separation of variables, while others concentrate on a wide range of topics including elliptic theory, Green's functions, variational, and numerical methods.

A rich collection of worked examples and exercises accompanies the text, along with a large number of illustrations and graphs to provide insight into the numerical examples. Solutions to selected exercises are included for students, while extended solution sets are available to lecturers from www.cambridge.org/9780521613231

"...a lively, applied, up-to-date presentation...These authors definitely display a mastery of the material presented, as well as outstanding exposition."

Robert E. O'Malley, Jr. University of Washington

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