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With the exception of Fig. [5.12] and [8.7], all the figures are original by the author. Software used: “Wolfram Mathematica® 11.3,” and “Inkscape® 0.92.”