

Table of Contents

Preface

v

1 Vectors and Matrices

1

1.1	Linear Combinations of Vectors	2
1.2	Lengths and Angles from Dot Products	11
1.3	Matrices and Column Spaces	20
1.4	Matrix Multiplication and $A = CR$	29

2 Solving Linear Equations $Ax = b$

39

2.1	The Idea of Elimination	40
2.2	Elimination Matrices and Inverse Matrices	49
2.3	Matrix Computations and $A = LU$	57
2.4	Permutations and Transposes	64

3 The Four Fundamental Subspaces

74

3.1	Vector Spaces and Subspaces	75
3.2	The Nullspace of A : Solving $Ax = 0$	83
3.3	The Complete Solution to $Ax = b$	96
3.4	Independence, Basis, and Dimension	107
3.5	Dimensions of the Four Subspaces	121

4 Orthogonality

134

4.1	Orthogonality of the Four Subspaces	135
4.2	Projections onto Subspaces	143
4.3	Least Squares Approximations	153
4.4	Orthogonal Matrices and Gram-Schmidt	165

5 Determinants and Linear Transformations

177

5.1	3 by 3 Determinants	178
5.2	Properties and Applications of Determinants	184
5.3	Linear Transformations	192

6 Eigenvalues and Eigenvectors

201

6.1	Introduction to Eigenvalues	202
6.2	Diagonalizing a Matrix	215
6.3	Symmetric Positive Definite Matrices	227
6.4	Systems of Differential Equations	243

7	The Singular Value Decomposition (SVD)	258
7.1	Singular Values and Singular Vectors	259
7.2	Compressing Images by the SVD	269
7.3	Principal Component Analysis	274
7.4	The Victory of Orthogonality (and a Revolution)	280
8	Learning from Data	286
8.1	Piecewise Linear Learning Functions	289
8.2	Convolutional Neural Nets	299
8.3	Minimizing Loss by Gradient Descent	306
8.4	Mean, Variance, and Covariance	321
Appendix 1	The Ranks of AB and $A + B$	334
Appendix 2	Eigenvalues and Singular Values : Rank One	335
Appendix 3	Counting Parameters in the Basic Factorizations	336
Appendix 4	Codes and Algorithms for Numerical Linear Algebra	337
Appendix 5	Matrix Factorizations	338
Appendix 6	The Column-Row Factorization of a Matrix	340
Appendix 7	The Jordan Form of a Square Matrix	343
Appendix 8	Tensors	344
Appendix 9	The Condition Number	345
Appendix 10	Markov Matrices and Perron-Frobenius	346
Index		348
Index of Symbols		354
Six Great Theorems / Linear Algebra in a Nutshell		356