Contents

5 Interval Graphs of Limited Nesting and Count of Lengths

This chapter contains:

- *5.1 Extending Partial Representations with Two Lengths.* We show an NP-hardness reduction for two lengths, which works even when they are known and each interval has its lenght prescribed by the input.
- *5.2 Basic Properties of k-nested Interval Graphs.* We introduce some definitions and describe an efficient encoding of k-nested interval graphs using $2n\lceil \log k + 1 \rceil$ bits. We present cleaned representations minimizing nesting for a consecutive ordering of maximal cliques and determine which nestings are forced in every interval representation.
- *5.3 Recognizing k-nested Interval Graphs.* We describe a linear-time algorithm for computing an interval representation of minimal nesting based on dynamic programming on MPQ-trees. We compute triples (α, β, γ) for each subtree of the MPQ-tree, from bottom to top, linked to minimal interval representations. Formulas for P-nodes and for Q-nodes are described, computing its triple from the triples of their subtrees.

http://pavel.klavik.cz/orgpad/nest_len_int.html

6 Overview of Algebraic Properties of Graphs

This chapter contains:

- *6.1: Outline.* We describe which algebraic properties are studied.
- *6.2: 3-connected Reduction.* We introduce the 3-connected reduction which generalizes connected components and block trees, and decomposes graphs into 3-connected components. All described results are proved in Chapter 7.
- *6.3: Automorphism Groups of Planar Graphs.* We describe results about automorphism groups of restricted classes of graphs and in particular of planar graphs. The inductive Jordan-like characterization of the automorphism groups of planar graphs is proved in Chapter 8.
- *6.4: Graph Isomorphism Problem.* Known complexity results are discussed.
- *6.5: Graph Isomorphism Problem Restricted by Lists.* Lubiw [260] proved that this generalization of the graph isomorphism problem is NP-complete. We describe results for variety of graph classes and graph parameters proved in Chapter 9.
- *6.6: Regular Graph Covers.* We describe motivations for regular graph covers, survey several related problems and describe structural results proved in Chapter 10 and algorithmic results proved in Chapter 11.

http://pavel.klavik.cz/orgpad/algebraic_properties.html

7 3-connected Reduction

This chapter contains:

- *7.1: Definition of Extended Graphs.* We define extended graphs with half-edges used in Part II.
- *7.2: Group Theory and Automorphism Groups of Graphs.* An overview of the main concepts from group theory.
- *7.3: Block Trees and Their Automorphisms.* Block trees decompose graphs into 2-connected subgraphs. They capture automorphism groups.
- *7.4: Structural Properties of Atoms.* We introduce atoms which are 3-connected components of the reduction.
- *7.5: Reduction Series and Reduction Trees.* We describe the reduction series which replaces atoms by colored edges. It is captured by the reduction tree.
- *7.6: Reduction Epimorphism.* We describe changes in automorphism groups by reductions, with a group epimorphism $\Phi_i : \mathrm{Aut}(G_i) \to \mathrm{Aut}(G_{i+1})$.
- *7.7: Polynomial-time Algorithms.* We describe algorithms for computing the reduction series and the reduction tree.

http://pavel.klavik.cz/orgpad/3conn_reduction.html

8 Jordan-like Characterization of Automorphism Groups of Planar Graphs

This chapter contains:

- *8.1: Automorphism Groups of 3-connected Planar Graphs.* We describe spherical groups and the geometry of automorphism groups of planar graphs. We also characterize automorphism groups of planar atoms and primitive graphs.
- *8.2: The Jordan-like Characterization.* We give the first inductive characterization of the automorphism groups of planar graphs. First, we characterize the stabilizers as the class of groups closed under 5 group products. Then, we combine the stabilizers with spherical groups.
- *8.3: Applications of Jordan-like Characterization.* We describe automorphism groups of 2-connected planar graphs, outerplanar graphs, and series-parallel graphs.
- *8.4: Comparison with Babai's Characterization.* We compare the Jordan-like inductive characterization with Babai's characterization [11] from 1975.
- *8.5: Quadratic-time Algorithm.* Our characterization implies a quadratic-time algorithm for computing automorphism groups of planar graphs.
- *8.6: Conclusions.* We describe open problems and a spacial visualization of the automorphism groups of planar graphs.

http://pavel.klavik.cz/orgpad/geom_aut_groups.html

9 Graph Isomorphism Restricted by Lists

This chapter contains:

- *9.1: Basic Results.* We describe bipartite perfect matchings and give basic algorithms for LISTISO.
- *9.2: GI-completeness of GraphIso Implies NP-completeness of ListIso.* We show that reduction for GRAPHISO using vertex-gadgets can be modified for LISTISO.
- *9.3: NP-completeness for 3-regular Colored Graphs.* We modify the reduction of Lubiw [260].
- *9.4, 9.5, and 9.6.* We describe combinatorial algorithms for GRAPHISO of trees, planar graphs, interval graphs, permutation graphs, and circle graphs, and modify them for LISTISO.
- *9.7 and 9.8.* We modify involved algorithms for GRAPHISO of bounded genus and bounded treewidth graphs to LISTISO.
- *9.9: Conclusions.* We describe related topics and open problems.

http://pavel.klavik.cz/orgpad/list_isomorphism.html

10 3-connected Reduction for Regular Graph Covers

This chapter contains:

- *10.1: Definition of Regular Graph Covering.* We define semiregular subgroups, regular covering projections and regular quotients.
- *10.2: Regular Projections and Quotients of Atoms.* We introduce halvable atoms and describe three types of quotients of atoms.
- *10.3: Quotient Expansions.* We revert the reductions in quotients and describe all quotients H_0 of G_0 rising from the quotients H_r of G_r.
- *10.4: Quotients of Planar Graphs and Negami's Theorem.* We apply the results to planar graphs. We describe all quotients of planar atoms and primitive graphs, and thus describe all quotients of planar graphs.
- *10.5: Concluding Remarks.* We conclude with remarks and open problems.

http://pavel.klavik.cz/orgpad/regular_covers.html

11 Algorithmic Aspects of Regular Graph Covers

This chapter contains:

- *11.1: Complexity of Regular Graph Covering.* We prove that REGULARCOVER belongs to NP and it is GI-hard.
- *11.2: Atoms, Reduction and Expansion.* We describe how to compute symmetry types and quotients of atoms.
- *11.3: Meta-algorithm.* We describe the FPT algorithm solving REGULARCOVER for graph classes satisfying (P1), (P2), and (P3). It is an involved dynamic programming on the reduction tree of H using two subroutines: a generalization of the perfect matching problem (the bottleneck) and LISTISO (fast).
- *11.4: Star Blocks Atoms with Lists.* We discuss details concerning the only slow subroutine in the meta-algorithm.
- *11.5: Applying the Meta-algorithm to Planar Graphs.* We show that planar graphs satisfy (P1), (P2), and (P3).
- *11.6: Concluding Remarks.* We describe the time analysis of the algorithm for regular covering testing of planar graphs and discuss open problems.

http://pavel.klavik.cz/orgpad/regular_covers.html

1 Introduction to Geometric Representations of Graphs

This chapter contains:

- *1.1: Motivation.* We introduce graph drawing and graph representations by three detailed motivations: 1) Tutte's spring embedding of planar graphs using linear algebra and physics, 2) Benzer's study of DNA using interval graphs, 3) circle packings of planar graphs and their relation to Riemann Mapping Theorem in complex analysis.
- *1.2: Definitions.* We define graphs, geometric representations, the recognition problems, and the other main definitions used in this thesis.
- *1.3: Intersection Representations.* We describe the main classes of intersection graphs which are discussed in this thesis.
- *1.4: Planar Embeddings.* We describe planar and spherical embeddings. Also, the special geometric role of 3-connected planar graphs due to Steinitz, Whitney and Mani is discussed.
- *1.5: Results of This Thesis.* We describe the organization of this thesis and give a short overview of the main proved results.

http://pavel.klavik.cz/orgpad/geom_rep.html

2 State of The Art for Partial Representation Extension

This chapter contains:

- *2.2 and 2.4.* An overview of results proved in Chapters 3, 4, and 5.
- *2.3, 2.5, 2.6 and 2.7.* We survey results for partial representation extension of proper interval, unit interval, chordal, circle, comparability, permutation, function, trapezoid, and proper-circular arc graphs.
- *2.8 Extending Other Types of Partial Representations.* We survey results for partial embedding extension, partial representation extension of contact representations of planar graphs, and partial visibility extension.
- *2.9 Related Restricted Representation Problems.* We describe chronological orderings, bounded representations, representation sandwich, simultaneous representations, and Allen algebras from time reasoning.
- *2.10 Open Problems.* We conclude with a list of open problems.

http://pavel.klavik.cz/orgpad/repext.html

3 Extending Partial Interval Representations in Linear Time

This chapter contains:

- *3.1: PQ-trees and Consecutive Orderings of Maximal Cliques.*
  Fulkerson and Gross [133] described that interval representations correspond to certain linear orderings of maximal cliques of an interval graph called consecutive orderings. Booth and Lueker [39] described a data structure called PQ-trees which efficiently stores all consecutive orderings.
- *3.2: Characterization of Extendible Partial Interval Representations.*
  We show that a partial representation $\mathcal{R}'$ gives a partial ordering $\lhd$ for maximal cliques such that it is extendible if and only if there exists a consecutive ordering of maximal cliques which extends $\lhd$.
- *3.3: The Reordering Problem of PQ-trees.* It asks whether a PQ-tree can be reordered to extend an input partial ordering. We describe two polynomial-time algorithms: one for general partial orderings and a faster one for interval orderings.
- *3.4: Linear-time Algorithm.* We construct a linear-time algorithm for RepExt(INT) by combining the above results.

http://pavel.klavik.cz/orgpad/repext_int.html

4 Minimal Obstructions for Partial Representation Extension of Interval Graphs

This chapter contains:

- *4.1: Definition of Minimal Obstructions.* We formally describe all minimal obstructions mentioned in Theorem 2.2.1 and prove that they are minimal and non-extendible.
- *4.2: MPQ-trees and Basic Tools.* We introduce MPQ-trees which are an augementation of PQ-trees described in Section 3.1. Also, we describe basic tools used in deriving minimal obstructions, namely Sliding Lemma.
- *4.3: Strategy for Finding Minimal Obstructions.* We describe our strategy for the proof that every non-extendible partial representation contains one of the described obstructions.
- *4.4, 4.5, and 4.6: Three Main Cases.* We divide the argument according to the type of obstructed node. The case of Q-nodes is most involved.
- *4.7: Proofs of Main Results.* We prove Theorem 2.2.1 and construct the linear-time certifying algorithm for RepExt(INT) of Theorem 2.2.3.
- *4.8: Conclusions.* We conclude with comments and several open problems.

http://pavel.klavik.cz/orgpad/minobstr.html