Contents

Acknowledgments xiii

Introduction xv

1 Cartesian Coordinate Systems 1
- 1.1 1D Mathematics . . . 2
- 1.2 2D Cartesian Space . . . 5
- 1.3 3D Cartesian Space . . . 12
- 1.4 Odds and Ends . . . 19
- 1.5 Exercises . . . 27

2 Vectors 31
- 2.1 Mathematical Definition of Vector, and Other Boring Stuff 32
- 2.2 Geometric Definition of Vector . . . 34
- 2.3 Specifying Vectors with Cartesian Coordinates . . . 36
- 2.4 Vectors versus Points . . . 39
- 2.5 Negating a Vector . . . 43
- 2.6 Vector Multiplication by a Scalar . . . 45
- 2.7 Vector Addition and Subtraction . . . 47
- 2.8 Vector Magnitude (Length) . . . 51
- 2.9 Unit Vectors . . . 53
- 2.10 The Distance Formula . . . 55
- 2.11 Vector Dot Product . . . 56
- 2.12 Vector Cross Product . . . 66
- 2.13 Linear Algebra Identities . . . 70
- 2.14 Exercises . . . 71

3 Multiple Coordinate Spaces 79
- 3.1 Why Bother with Multiple Coordinate Spaces? . . . 80
- 3.2 Some Useful Coordinate Spaces . . . 81
- 3.3 Basis Vectors and Coordinate Space Transformations . . 86
- 3.4 Nested Coordinate Spaces . . . 106

3.5 In Defense of Upright Space . . . 108
3.6 Exercises . . . 109

4 Introduction to Matrices 113
4.1 Mathematical Definition of Matrix . . . 113
4.2 Geometric Interpretation of Matrix . . . 124
4.3 The Bigger Picture of Linear Algebra . . . 130
4.4 Exercises . . . 132

5 Matrices and Linear Transformations 137
5.1 Rotation . . . 138
5.2 Scale . . . 144
5.3 Orthographic Projection . . . 148
5.4 Reflection . . . 151
5.5 Shearing . . . 152
5.6 Combining Transformations . . . 153
5.7 Classes of Transformations . . . 154
5.8 Exercises . . . 159

6 More on Matrices 161
6.1 Determinant of a Matrix . . . 161
6.2 Inverse of a Matrix . . . 168
6.3 Orthogonal Matrices . . . 171
6.4 4×4 Homogeneous Matrices . . . 176
6.5 4×4 Matrices and Perspective Projection . . . 183
6.6 Exercises . . . 189

7 Polar Coordinate Systems 191
7.1 2D Polar Space . . . 191
7.2 Why Would Anybody Use Polar Coordinates? . . . 201
7.3 3D Polar Space . . . 203
7.4 Using Polar Coordinates to Specify Vectors . . . 213
7.5 Exercises . . . 214

8 Rotation in Three Dimensions 217
8.1 What Exactly is "Orientation"? . . . 218
8.2 Matrix Form . . . 220
8.3 Euler Angles . . . 229
8.4 Axis-Angle and Exponential Map Representations . . . 244
8.5 Quaternions . . . 246
8.6 Comparison of Methods . . . 273
8.7 Converting between Representations . . . 275
8.8 Exercises . . . 291

9 Geometric Primitives 295
9.1 Representation Techniques . . . 295
9.2 Lines and Rays . . . 297
9.3 Spheres and Circles . . . 303
9.4 Bounding Boxes . . . 304
9.5 Planes . . . 311
9.6 Triangles . . . 317
9.7 Polygons . . . 332
9.8 Exercises . . . 339

10 Mathematical Topics from 3D Graphics 343
10.1 How Graphics Works . . . 345
10.2 Viewing in 3D . . . 362
10.3 Coordinate Spaces . . . 369
10.4 Polygon Meshes . . . 381
10.5 Texture Mapping . . . 393
10.6 The Standard Local Lighting Model . . . 396
10.7 Light Sources . . . 414
10.8 Skeletal Animation . . . 424
10.9 Bump Mapping . . . 431
10.10 The Real-Time Graphics Pipeline . . . 438
10.11 Some HLSL Examples . . . 457
10.12 Further Reading . . . 475
10.13 Exercises . . . 476

11 Mechanics 1: Linear Kinematics and Calculus 479
11.1 Overview and Other Expectation-Reducing Remarks . . . 479
11.2 Basic Quantities and Units . . . 483
11.3 Average Velocity . . . 486
11.4 Instantaneous Velocity and the Derivative . . . 490
11.5 Acceleration . . . 513
11.6 Motion under Constant Acceleration . . . 516
11.7 The Integral . . . 530
11.8 Uniform Circular Motion . . . 542
11.9 Exercises . . . 549

12 Mechanics 2: Linear and Rotational Dynamics 553
12.1 Newton's Three Laws . . . 554
12.2 Some Simple Force Laws . . . 562
12.3 Momentum . . . 581
12.4 Impulsive Forces and Collisions . . . 590
12.5 Rotational Dynamics . . . 603
12.6 Real-Time Rigid Body Simulators . . . 621

12.7 Suggested Reading 639
12.8 Exercises 640

13 Curves in 3D 645
13.1 Parametric Polynomial Curves 646
13.2 Polynomial Interpolation 653
13.3 Hermite Curves 665
13.4 Bézier Curves 670
13.5 Subdivision 685
13.6 Splines 690
13.7 Hermite and Bézier Splines 694
13.8 Continuity 697
13.9 Automatic Tangent Control 702
13.10 Exercises 711

14 Afterword 715
14.1 What Next? 715
14.2 Exercises 715

A Geometric Tests 717
A.1 Closest Point on 2D Implicit Line 717
A.2 Closest Point on a Parametric Ray 718
A.3 Closest Point on a Plane 719
A.4 Closest Point on a Circle or Sphere 719
A.5 Closest Point in an AABB 720
A.6 Intersection Tests 720
A.7 Intersection of Two Implicit Lines in 2D 721
A.8 Intersection of Two Rays in 3D 722
A.9 Intersection of a Ray and Plane 724
A.10 Intersection of an AABB and Plane 725
A.11 Intersection of Three Planes 726
A.12 Intersection of Ray and a Circle or Sphere 727
A.13 Intersection of Two Circles or Spheres 729
A.14 Intersection of a Sphere and AABB 731
A.15 Intersection of a Sphere and a Plane 732
A.16 Intersection of a Ray and a Triangle 734
A.17 Intersection of Two AABBs 737
A.18 Intersection of a Ray and an AABB 740

B Answers to the Exercises 745
B.1 Chapter 1 745
B.2 Chapter 2 746
B.3 Chapter 3 758

B.4 Chapter 4 759
B.5 Chapter 5 763
B.6 Chapter 6 765
B.7 Chapter 7 767
B.8 Chapter 8 772
B.9 Chapter 9 774
B.10 Chapter 10 779
B.11 Chapter 11 781
B.12 Chapter 12 784
B.13 Chapter 13 792
B.14 Chapter 14 799

Bibliography 801

Index 807

So much time, and so little to do!
Strike that, reverse it.

— Willy Wonka