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Recent innovation in 3D acquisition technology, such as computer tomography, magnetic resonance imaging, 3D laser scanning, ultrasound, radar, and microscopy has enabled highly accurate digitization of complex 3D objects. Numerous scientific disciplines, such as neuroscience, mechanical engineering, and astrophysics, rely on the analysis and processing of such geometric data to understand intricate geometric structures and facilitate new scientific discoveries. A similar abundance of digital 3D content can be observed in other fields and industries, including entertainment, cultural heritage, geo-exploration, architecture, and urban modeling. Concurrent to these advances in 3D sensing technology, we are experiencing a revolution in digital manufacturing technology (e.g., in bio-medicine, commodity product design, and architecture). Novel materials and robotic production will soon allow the automated creation of complex, fully functional physical artifacts from a digital design plan.

Between acquisition and production lies the discipline of *digital geometry processing*, a relatively new field of computer science that is concerned with mathematical models and algorithms for analyzing and manipulating geometric data. Typical operations include surface reconstruction from point samples, filtering operations for noise removal, geometry analysis, shape simplification, and geometric modeling and interactive design. The abundance of data sources, processing operations, and manufacturing technologies has resulted in a great wealth of mathematical representations for geometric data. In this context, polygon meshes have become increasingly popular in recent years and are nowadays used intensively in