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The main outline of the text has not been changed. The first chapter is an informal discussion of set theory. The concept of equivalence relations has been postponed until Chapter 3, where it appears in the context of compactness.

The second chapter is a discussion of metric spaces. The topological terms "open set," "neighborhood," etc., are introduced. Particular attention is paid to various distance functions which may be defined on Euclidean n -space and which lead to the ordinary topology.

In taking up topological spaces in Chapter 3, the transition from the particular to the general has been maintained so that the concept of topological space is viewed as a generalization of the concept of metric space. Thus there is a similarity or, perhaps, a redundancy in the presentation of these two topics. A great deal of attention has been paid to alternate procedures for the creation of a topological space, using gauge spaces, topological groups, and neighborhood systems, etc., in the hope that this seemingly trivial, but subtle, point would be clarified.

Chapters 4 and 5 are devoted to a discussion of the two most important topological properties, connectedness and compactness. Some of this material would lead to further discussion of topics related to analysis, function spaces, separation axioms, metrization theorems, to name a few. On the other hand, such as homotopy and two-point compactifications are also included, but these are treated in a separate chapter to avoid further delay in the further discussion of topology.

In conclusion, it is a pleasure to express my gratitude to the many individuals who have helped me in the preparation of this book. In particular, I should like to mention Professors C. Chevalley, S. Eilenberg, I. James, H. Ribes, P. Smith, and R. Thomas.

R. M.