Contents

1 Introduction **1**
1.1 Modeling Languages for Optimization 1
1.2 Modeling with Pyomo 3
1.2.1 Simple Examples 3
1.2.2 Graph Coloring Example 5
1.2.3 Key Pyomo Features 6
1.3 Getting Started 9
1.4 Book Summary 10
1.5 Discussion 10

Part I An Introduction to Pyomo **13**

2 Mathematical Modeling and Optimization **15**
2.1 Mathematical Modeling 15
2.1.1 Overview 15
2.1.2 A Modeling Example 16
2.2 Optimization 18
2.3 Modeling with Pyomo 20
2.3.1 A Concrete Formulation 20
2.4 Linear and Nonlinear Optimization Models 22
2.4.1 Definition 22
2.4.2 Linear Version 23
2.5 Solving the Pyomo Model 23
2.5.1 Solvers 23
2.5.2 Python Scripts 23

3 Pyomo Overview **25**
3.1 Introduction 25
3.2 The Warehouse Location Problem 26
3.3 Pyomo Models 27
3.3.1 Components for Variables, Objectives, and Constraints . . 27
3.3.2 Indexed Components 28
3.3.3 Construction Rules 30
3.3.4 A Concrete Model for the Warehouse Location Problem . 31

3.3.5 Modeling Components for Sets and Parameters 34

4 Pyomo Models and Components: An Introduction **37**
4.1 An Object-Oriented AML 37
4.2 Common Component Paradigms 38
4.2.1 Indexed Components 39
4.3 Variables 40
4.3.1 `Var` Declarations 40
4.3.2 Working with `Var` Objects 43
4.4 Objectives 43
4.4.1 `Objective` Declarations 44
4.4.2 Working with `Objective` Objects 45
4.5 Constraints 46
4.5.1 `Constraint` Declarations 46
4.5.2 Working with `Constraint` Objects 48
4.6 Set Data 49
4.6.1 `Set` Declarations 49
4.6.2 Working with `Set` Objects 52
4.7 Parameter Data 54
4.7.1 `Param` Declarations 55
4.7.2 Working with `Param` Objects 58
4.8 Named Expressions 59
4.8.1 `Expression` Declarations 59
4.8.2 Working with `Expression` Objects 60
4.9 Suffix Components 61
4.9.1 `Suffix` Declarations 61
4.9.2 Working with Suffixes 63
4.10 Other Modeling Components 64

5 Scripting Custom Workflows **67**
5.1 Introduction 67
5.2 Interrogating the Model 70
5.2.1 The `value` Function 71
5.2.2 Accessing Attributes of Indexed Components 72
5.3 Modifying Pyomo Model Structure 73
5.4 Examples of Common Scripting Tasks 74
5.4.1 Warehouse Location Loop and Plotting 74
5.4.2 A Sudoku Solver 76

6 Interacting with Solvers **83**
6.1 Introduction 83
6.2 Using Solvers 84
6.3 Investigating the Solution 86
6.3.1 Solver Results 86

Part II Advanced Topics **89**

7 Nonlinear Programming with Pyomo **91**
7.1 Introduction 91
7.2 Nonlinear Progamming Problems in Pyomo 92
7.2.1 Nonlinear Expressions 92
7.2.2 The Rosenbrock Problem 93
7.3 Solving Nonlinear Programming Formulations 95
7.3.1 Nonlinear Solvers 96
7.3.2 Additional Tips for Nonlinear Programming 96
7.4 Nonlinear Programming Examples 98
7.4.1 Variable Initialization for a Multimodal Function 98
7.4.2 Optimal Quotas for Sustainable Harvesting of Deer 99
7.4.3 Estimation of Infectious Disease Models 103
7.4.4 Reactor Design 106

8 Structured Modeling with Blocks **111**
8.1 Introduction 111
8.2 Block structures 113
8.3 Blocks as Indexed Components 115
8.4 Construction Rules within Blocks 116
8.5 Extracting values from hierarchical models 117
8.6 Blocks Example: Optimal Multi-Period Lot-Sizing 117
8.6.1 A Formulation Without Blocks 119
8.6.2 A Formulation With Blocks 120

9 Performance: Model Construction and Solver Interfaces **123**
9.1 Profiling to Identify Performance Bottlenecks 123
9.1.1 Report Timing 125
9.1.2 TicTocTimer 125
9.1.3 Profilers 126
9.2 Improving Model Construction Performance with LinearExpression 128
9.3 Repeated Solves with Persistent Solvers 129
9.3.1 When to Use a Persistent Solver 130
9.3.2 Basic Usage 130
9.3.3 Working with Indexed Variables and Constraints 132
9.3.4 Additional Performance 132
9.3.5 Example 133
9.4 Sparse Index Sets 134

10 Abstract Models and Their Solution **137**
10.1 Overview 137
10.1.1 Abstract and Concrete Models 137
10.1.2 An Abstract Formulation of Model (H) 139
10.1.3 An Abstract Model for the Warehouse Location Problem . 141
10.2 The `pyomo` Command 143

10.2.1 The `help` Subcommand . . . 144
10.2.2 The `solve` Subcommand . . . 144
10.2.3 The `convert` Subcommand . . . 157
10.3 Data Commands for `AbstractModel` . . . 158
10.3.1 The `set` Command . . . 159
10.3.2 The `param` Command . . . 161
10.3.3 The `include` Command . . . 166
10.3.4 Data Namespaces . . . 166
10.4 Build Components . . . 166

Part III Modeling Extensions **169**

11 Generalized Disjunctive Programming **171**
11.1 Introduction . . . 171
11.2 Modeling GDP in Pyomo . . . 174
11.3 Expressing logical constraints . . . 176
11.4 Solving GDP models . . . 177
11.4.1 Big-M transformation . . . 177
11.4.2 Hull transformation . . . 178
11.5 A mixing problem with semi-continuous variables . . . 179

12 Differential Algebraic Equations **181**
12.1 Introduction . . . 181
12.2 Pyomo DAE Modeling Components . . . 182
12.3 Solving Pyomo Models with DAEs . . . 184
12.3.1 Finite Difference Transformation . . . 185
12.3.2 Collocation Transformation . . . 186
12.4 Additional Features . . . 187
12.4.1 Applying Multiple Discretizations . . . 187
12.4.2 Restricting Control Input Profiles . . . 188
12.4.3 Plotting . . . 188

13 Mathematical Programs with Equilibrium Constraints **191**
13.1 Introduction . . . 191
13.2 Modeling Equilibrium Conditions . . . 192
13.2.1 Complementarity Conditions . . . 192
13.2.2 Complementarity Expressions . . . 193
13.2.3 Modeling Mixed-Complementarity Conditions . . . 193
13.3 MPEC Transformations . . . 196
13.3.1 Standard Form . . . 197
13.3.2 Simple Nonlinear . . . 198
13.3.3 Simple Disjunction . . . 198
13.3.4 AMPL Solver Interface . . . 199
13.4 Solver Interfaces and Meta-Solvers . . . 200
13.4.1 Nonlinear Reformulations . . . 200

13.4.2 Disjunctive Reformulations 201
13.4.3 PATH and the ASL Solver Interface 201
13.5 Discussion 202

A A Brief Python Tutorial **203**
A.1 Overview 203
A.2 Installing and Running Python 204
A.3 Python Line Format 205
A.4 Variables and Data Types 206
A.5 Data Structures 207
A.5.1 Strings 207
A.5.2 Lists 207
A.5.3 Tuples 208
A.5.4 Sets 209
A.5.5 Dictionaries 209
A.6 Conditionals 210
A.7 Iterations and Looping 210
A.8 Generators and List Comprehensions 211
A.9 Functions 211
A.10 Objects and Classes 213
A.11 Assignment, `copy` and `deepcopy` 214
A.11.1 References 214
A.11.2 Copying 215
A.12 Modules 215
A.13 Python Resources 216

Bibliography **217**

Index **221**