

Contents

List of Symbols	xvii
Preface to the Second Edition	xxiii
Preface to the First Edition	xxv
About the Author	xxvii
1 INTRODUCTION	1
1.1 Continuum Mechanics	1
1.2 A Look Forward	4
1.3 Summary	5
Problems	6
2 VECTORS AND TENSORS	9
2.1 Background and Overview	9
2.2 Vector Algebra	10
2.2.1 Definition of a Vector	10
2.2.1.1 Vector addition	11
2.2.1.2 Multiplication of a vector by a scalar	11
2.2.1.3 Linear independence of vectors	11
2.2.2 Scalar and Vector Products	12
2.2.2.1 Scalar product	12
2.2.2.2 Vector product	13
2.2.2.3 Triple products of vectors	16
2.2.3 Plane Area as a Vector	17
2.2.4 Reciprocal Basis	19
2.2.4.1 Components of a vector	19
2.2.4.2 General basis	19
2.2.4.3 Orthonormal basis	21
2.2.4.4 The Gram–Schmidt orthonormalization	22
2.2.5 Summation Convention	23
2.2.5.1 Dummy index	24
2.2.5.2 Free index	24
2.2.5.3 Kronecker delta	25
2.2.5.4 Permutation symbol	25
2.2.6 Transformation Law for Different Bases	28
2.2.6.1 General transformation laws	28
2.2.6.2 Transformation laws for orthonormal systems	29

2.3 Theory of Matrices	31
2.3.1 Definition	31
2.3.2 Matrix Addition and Multiplication of a Matrix by a Scalar	32
2.3.3 Matrix Transpose	33
2.3.4 Symmetric and Skew Symmetric Matrices	33
2.3.5 Matrix Multiplication	34
2.3.6 Inverse and Determinant of a Matrix	36
2.3.7 Positive-Definite and Orthogonal Matrices	39
2.4 Vector Calculus	40
2.4.1 Differentiation of a Vector with Respect to a Scalar	40
2.4.2 Curvilinear Coordinates	42
2.4.3 The Fundamental Metric	43
2.4.4 Derivative of a Scalar Function of a Vector	44
2.4.5 The Del Operator	45
2.4.6 Divergence and Curl of a Vector	47
2.4.7 Cylindrical and Spherical Coordinate Systems	51
2.4.8 Gradient, Divergence, and Curl Theorems	52
2.5 Tensors	53
2.5.1 Dyads and Dyadics	53
2.5.2 Nonion Form of a Second-Order Tensor	54
2.5.3 Transformation of Components of a Tensor	57
2.5.4 Higher-Order Tensors	58
2.5.5 Tensor Calculus	59
2.5.6 Eigenvalues and Eigenvectors	62
2.5.6.1 Eigenvalue problem	62
2.5.6.2 Eigenvalues and eigenvectors of a real symmetric tensor	62
2.5.6.3 Spectral theorem	64
2.5.6.4 Calculation of eigenvalues and eigenvectors	64
2.6 Summary	72
Problems	73
3 KINEMATICS OF CONTINUA	81
3.1 Introduction	81

3.2 Descriptions of Motion	82
3.2.1 Configurations of a Continuous Medium	82
3.2.2 Material Description	83
3.2.3 Spatial Description	85
3.2.4 Displacement Field	88
3.3 Analysis of Deformation	89
3.3.1 Deformation Gradient	89
3.3.2 Isochoric, Homogeneous, and Inhomogeneous Deformations	93
3.3.2.1 Isochoric deformation	93
3.3.2.2 Homogeneous deformation	93
3.3.2.3 Nonhomogeneous deformation	95
3.3.3 Change of Volume and Surface	96
3.3.3.1 Volume change	96
3.3.3.2 Area change	97
3.4 Strain Measures	98
3.4.1 Cauchy–Green Deformation Tensors	98
3.4.2 Green–Lagrange Strain Tensor	100
3.4.3 Physical Interpretation of Green–Lagrange Strain Components	101
3.4.4 Cauchy and Euler Strain Tensors	103
3.4.5 Transformation of Strain Components	106
3.4.6 Invariants and Principal Values of Strains	109
3.5 Infinitesimal Strain Tensor and Rotation Tensor	111
3.5.1 Infinitesimal Strain Tensor	111
3.5.2 Physical Interpretation of Infinitesimal Strain Tensor Components	112
3.5.3 Infinitesimal Rotation Tensor	114
3.5.4 Infinitesimal Strains in Cylindrical and Spherical Coordinate Systems	116
3.5.4.1 Cylindrical coordinate system	117
3.5.4.2 Spherical coordinate system	117
3.6 Velocity Gradient and Vorticity Tensors	118
3.6.1 Definitions	118
3.6.2 Relationship Between \mathbf{D} and $\dot{\mathbf{E}}$	119

3.7 Compatibility Equations	120
3.7.1 Preliminary Comments	120
3.7.2 Infinitesimal Strains	121
3.7.3 Finite Strains	125
3.8 Rigid-Body Motions and Material Objectivity	125
3.8.1 Superposed Rigid-Body Motions	125
3.8.1.1 Introduction and rigid-body transformation	125
3.8.1.2 Effect on \mathbf{F}	128
3.8.1.3 Effect on \mathbf{C} and \mathbf{E}	128
3.8.1.4 Effect on \mathbf{L} and \mathbf{D}	129
3.8.2 Material Objectivity	129
3.8.2.1 Observer transformation	129
3.8.2.2 Objectivity of various kinematic measures	130
3.8.2.3 Time rate of change in a rotating frame of reference	131
3.9 Polar Decomposition Theorem	132
3.9.1 Preliminary Comments	132
3.9.2 Rotation and Stretch Tensors	132
3.9.3 Objectivity of Stretch Tensors	138
3.10 Summary	139
Problems	140
4 STRESS MEASURES	151
4.1 Introduction	151
4.2 Cauchy Stress Tensor and Cauchy's Formula	151
4.2.1 Stress Vector	151
4.2.2 Cauchy's Formula	152
4.2.3 Cauchy Stress Tensor	153
4.3 Transformation of Stress Components and Principal Stresses	157
4.3.1 Transformation of Stress Components	157
4.3.1.1 Invariants	157
4.3.1.2 Transformation equations	157
4.3.2 Principal Stresses and Principal Planes	160
4.3.3 Maximum Shear Stress	162

4.4 Other Stress Measures	164
4.4.1 Preliminary Comments	164
4.4.2 First Piola–Kirchhoff Stress Tensor	164
4.4.3 Second Piola–Kirchhoff Stress Tensor	165
4.5 Equilibrium Equations for Small Deformations	169
4.6 Objectivity of Stress Tensors	171
4.6.1 Cauchy Stress Tensor	171
4.6.2 First Piola–Kirchhoff Stress Tensor	172
4.6.3 Second Piola–Kirchhoff Stress Tensor	172
4.7 Summary	172
Problems	173
5 CONSERVATION AND BALANCE LAWS	181
5.1 Introduction	181
5.2 Conservation of Mass	182
5.2.1 Preliminary Discussion	182
5.2.2 Material Time Derivative	182
5.2.3 Vector and Integral Identities	184
5.2.3.1 Vector identities	184
5.2.3.2 Integral identities	185
5.2.4 Continuity Equation in the Spatial Description	185
5.2.5 Continuity Equation in the Material Description	191
5.2.6 Reynolds Transport Theorem	193
5.3 Balance of Linear and Angular Momentum	193
5.3.1 Principle of Balance of Linear Momentum	193
5.3.1.1 Equations of motion in the spatial description	197
5.3.1.2 Equations of motion in the material description	199
5.3.2 Spatial Equations of Motion in Cylindrical and Spherical Coordinates	201
5.3.2.1 Cylindrical coordinates	202
5.3.2.2 Spherical coordinates	202
5.3.3 Principle of Balance of Angular Momentum	203
5.3.3.1 Monopolar case	203
5.3.3.2 Multipolar case	205

5.4 Thermodynamic Principles	206
5.4.1 Introduction	206
5.4.2 Balance of Energy	207
5.4.2.1 Energy equation in the spatial description	207
5.4.2.2 Energy equation in the material description	209
5.4.3 Entropy Inequality	210
5.4.3.1 Homogeneous processes	210
5.4.3.2 Inhomogeneous processes	210
5.5 Summary	212
5.5.1 Preliminary Comments	212
5.5.2 Conservation and Balance Equations in the Spatial Description	212
5.5.3 Conservation and Balance Equations in the Material Description	213
5.5.4 Closing Comments	213
Problems	214
6 CONSTITUTIVE EQUATIONS	221
6.1 Introduction	221
6.1.1 General Comments	221
6.1.2 General Principles of Constitutive Theory	222
6.1.3 Material Frame Indifference	223
6.1.4 Restrictions Placed by the Entropy Inequality	224
6.2 Elastic Materials	225
6.2.1 Cauchy-Elastic Materials	225
6.2.2 Green-Elastic or Hyperelastic Materials	226
6.2.3 Linearized Hyperelastic Materials: Infinitesimal Strains	227
6.3 Hookean Solids	228
6.3.1 Generalized Hooke's Law	228
6.3.2 Material Symmetry Planes	230
6.3.3 Monoclinic Materials	232
6.3.4 Orthotropic Materials	233
6.3.5 Isotropic Materials	237
6.4 Nonlinear Elastic Constitutive Relations	241

6.5 Newtonian Fluids	242
6.5.1 Introduction	242
6.5.2 Ideal Fluids	243
6.5.3 Viscous Incompressible Fluids	244
6.6 Generalized Newtonian Fluids	245
6.6.1 Introduction	245
6.6.2 Inelastic Fluids	245
6.6.2.1 Power-law model	246
6.6.2.2 Carreau model	246
6.6.2.3 Bingham model	247
6.6.3 Viscoelastic Constitutive Models	247
6.6.3.1 Differential models	247
6.6.3.2 Integral models	250
6.7 Heat Transfer	251
6.7.1 Introduction	251
6.7.2 Fourier's Heat Conduction Law	251
6.7.3 Newton's Law of Cooling	252
6.7.4 Stefan–Boltzmann Law	252
6.8 Constitutive Relations for Coupled Problems	252
6.8.1 Electromagnetics	252
6.8.1.1 Maxwell's equations	253
6.8.1.2 Constitutive relations	253
6.8.2 Thermoelasticity	255
6.8.3 Hygrothermal elasticity	255
6.8.4 Electroelasticity	256
6.9 Summary	258
Problems	259
7 LINEARIZED ELASTICITY	265
7.1 Introduction	265
7.2 Governing Equations	265
7.2.1 Preliminary Comments	266
7.2.2 Summary of Equations	266

7.2.2.1 Strain-displacement equations	266
7.2.2.2 Equations of motion	267
7.2.2.3 Constitutive equations	268
7.2.2.4 Boundary conditions	269
7.2.2.5 Compatibility conditions	269
7.2.3 The Navier Equations	269
7.2.4 The Beltrami–Michell Equations	270
7.3 Solution Methods	271
7.3.1 Types of Problems	271
7.3.2 Types of Solution Methods	272
7.3.3 Examples of the Semi-Inverse Method	273
7.3.4 Stretching and Bending of Beams	278
7.3.5 Superposition Principle	283
7.3.6 Uniqueness of Solutions	284
7.4 Clapeyron’s, Betti’s, and Maxwell’s Theorems	285
7.4.1 Clapeyron’s Theorem	285
7.4.2 Betti’s Reciprocity Theorem	288
7.4.3 Maxwell’s Reciprocity Theorem	291
7.5 Solution of Two-Dimensional Problems	293
7.5.1 Introduction	293
7.5.2 Plane Strain Problems	294
7.5.3 Plane Stress Problems	297
7.5.4 Unification of Plane Stress and Plane Strain Problems	300
7.5.5 Airy Stress Function	301
7.5.6 Saint-Venant’s Principle	303
7.5.7 Torsion of Cylindrical Members	308
7.5.7.1 Warping function	309
7.5.7.2 Prandtl’s stress function	311
7.6 Methods Based on Total Potential Energy	314
7.6.1 Introduction	314
7.6.2 The Variational Operator	314
7.6.3 The Principle of the Minimum Total Potential Energy	316
7.6.3.1 Construction of the total potential energy functional	316
7.6.3.2 Euler’s equations and natural boundary conditions	317
7.6.3.3 Minimum property of the total potential energy functional	319

7.6.4 Castigliano's Theorem I	322
7.6.5 The Ritz Method	326
7.6.5.1 The variational problem	326
7.6.5.2 Description of the method	328
7.7 Hamilton's Principle	334
7.7.1 Introduction	334
7.7.2 Hamilton's Principle for a Rigid Body	334
7.7.3 Hamilton's Principle for a Continuum	338
7.8 Summary	341
Problems	342
8 FLUID MECHANICS AND HEAT TRANSFER	355
8.1 Governing Equations	355
8.1.1 Preliminary Comments	355
8.1.2 Summary of Equations	356
8.2 Fluid Mechanics Problems	357
8.2.1 Governing Equations of Viscous Fluids	357
8.2.2 Inviscid Fluid Statics	360
8.2.3 Parallel Flow (Navier–Stokes Equations)	362
8.2.4 Problems with Negligible Convective Terms	367
8.2.5 Energy Equation for One-Dimensional Flows	370
8.3 Heat Transfer Problems	373
8.3.1 Governing Equations	373
8.3.2 Heat Conduction in a Cooling Fin	374
8.3.3 Axisymmetric Heat Conduction in a Circular Cylinder	376
8.3.4 Two-Dimensional Heat Transfer	379
8.3.5 Coupled Fluid Flow and Heat Transfer	381
8.4 Summary	382
Problems	382
9 LINEARIZED VISCOELASTICITY	389
9.1 Introduction	389
9.1.1 Preliminary Comments	389

9.1.2 Initial Value Problem, the Unit Impulse, and the Unit Step Function	390
9.1.3 The Laplace Transform Method	392
9.2 Spring and Dashpot Models	396
9.2.1 Creep Compliance and Relaxation Modulus	396
9.2.2 Maxwell Element	397
9.2.2.1 Creep response	397
9.2.2.2 Relaxation response	398
9.2.3 Kelvin–Voigt Element	400
9.2.3.1 Creep response	400
9.2.3.2 Relaxation response	401
9.2.4 Three-Element Models	402
9.2.5 Four-Element Models	404
9.3 Integral Constitutive Equations	407
9.3.1 Hereditary Integrals	407
9.3.2 Hereditary Integrals for Deviatoric Components	410
9.3.3 The Correspondence Principle	412
9.3.4 Elastic and Viscoelastic Analogies	414
9.4 Summary	420
Problems	420
References for Additional Reading	425
Answers to Selected Problems	429
Index	441