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Proofs of Impossibility, Proofs of Nonexistence

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The reason we start with proofs of impossibility is that they provide the best	
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