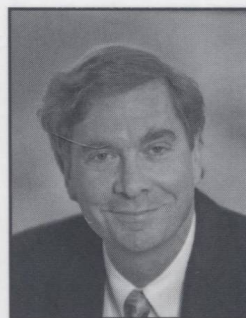


This book is derived from lecture notes for a course on Fourier analysis for engineering and science students at the advanced undergraduate or beginning graduate level. Beyond teaching specific topics and techniques—all of which are important in many areas of engineering and science—the author's goal is to help engineering and science students cultivate more advanced mathematical know-how and increase confidence in learning and using mathematics, as well as appreciate the coherence of the subject. He promises the readers a little magic on every page.



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The section headings are all recognizable to mathematicians, but the arrangement and emphasis are directed toward students from other disciplines. The material also serves as a foundation for advanced courses in signal processing and imaging. There are over 200 problems, many of which are oriented to applications, and a number use standard software. An unusual feature for courses meant for engineers is a more detailed and accessible treatment of distributions and the generalized Fourier transform. There is also more coverage of higher-dimensional phenomena than is found in most books at this level.

A thoroughly enjoyable yet careful mathematical perspective of the underlying concepts and many applications of modern signal analysis.

—Les Atlas, University of Washington

Osgood leads his readers from the basics to the more sophisticated parts of applicable Fourier analysis with a lively style, a light touch on the technicalities, and an eye toward communications engineering. This book should be a great resource for students of mathematics, physics, and engineering alike.

—Gerald B. Folland, University of Washington

Fourier analysis with a swing in its step.

—Tom Körner, University of Cambridge

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